TABLES AND FORMULAS

LONGLEY

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TABLES AND FORMULAS

FOR SOLVING NUMERICAL PROBLEMS IN
ANALYTIC GEOMETRY, CALCULUS
AND APPLIED MATHEMATICS

ARRANGED BY

WILLIAM RAYMOND LONGLEY, PH.D.

ASSISTANT PROFESSOR OF MATHEMATICS IN
THE SHEFFIELD SCIENTIFIC SCHOOL, YALE UNIVERSITY

REVISED EDITION

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PREFACE

This collection of tables and formulas is intended for use as a handbook for solving numerical problems in connection with the courses in mathematics in technical schools and colleges.

The numerical tables are those which have been found by experience to be most useful in performing the calculations necessary for careful plotting of various algebraic and transcendental curves in rectangular and polar coördinates (including parametric representation), for the approximate solution of equations, particularly transcendental equations, and for many types of problems in calculus and mechanics. Much emphasis is being given to numerical problem work illustrating the mathematical theory in engineering schools, and it is believed the student should have every time-saving facility for obtaining numerical results of a given degree of accuracy commensurate with that required in practical work. For this purpose the more extended tables are usually too cumbersome.

The formulas are selected from algebra, trigonometry, geometry, and calculus (including a short table of integrals), and the collection includes those that have been found useful in the courses in mathematics in the Sheffield Scientific School. It is not intended to provide the student with a compendium of all formulas that he may require, so that problem work becomes mere substitution of numbers in equations, but to give those which save time and relieve the burden on the memory. Following this idea certain formulas have been

omitted; for example, the mass of a body of variable density, center of gravity, and moment of inertia. It is impossible to apply these formulas to a practical problem without a thorough understanding of the fundamental theorem of the Integral Calculus. If this theorem has been mastered the formulas are unnecessary.

W. R. LONGLEY

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TABLES AND FORMULAS

NUMERICAL CONSTANTS

$$\begin{array}{ll} \pi = 3.14159. & \frac{1}{\pi^2} = 0.10132. \\ \pi^2 = 9.86960. & \frac{1}{\sqrt{\pi}} = 0.56419. \\ \sqrt{\pi} = 1.77245. & \log_{10} \pi = 0.49715. \\ \frac{1}{\pi} = 0.31831. & \log_e \pi = 1.14473. \end{array}$$

$$\begin{aligned} 1 \text{ radian} &= 57.296 \text{ degrees.} \\ &= 3437.75 \text{ minutes.} \\ &= 206264.8 \text{ seconds.} \end{aligned}$$

$$0.1 \text{ radian} = 5^\circ 43' 46.5''.$$

$$0.01 \text{ radian} = 0^\circ 34' 22.6''.$$

$$1 \text{ degree} = 0.01745 \text{ radian.}$$

$$1 \text{ minute} = 0.00029 \text{ radian.}$$

$$e \text{ (Napierian base)} = 2.71828.$$

$$\log_e 10 = M = 2.30259.$$

$$\log_{10} e = \frac{1}{M} = 0.43429.$$

Angle	sin	cos	tan	cot	sec	csc
0	0	1	0	∞	1	∞
30°	$\frac{1}{2}$	$\frac{1}{2}\sqrt{3}$	$\frac{1}{3}\sqrt{3}$	$\sqrt{3}$	$\frac{2}{3}\sqrt{3}$	2
45°	$\frac{1}{2}\sqrt{2}$	$\frac{1}{2}\sqrt{2}$	1	1	$\sqrt{2}$	$\sqrt{2}$
60°	$\frac{1}{2}\sqrt{3}$	$\frac{1}{2}$	$\sqrt{3}$	$\frac{1}{3}\sqrt{3}$	2	$\frac{2}{3}\sqrt{3}$
90°	1	0	∞	0	∞	1

N.	0	1	2	3	4	5	6	7	8	9	P.P.		
0	—	0000	3010	4771	6021	6990	7782	8451	9031	9542		22	21
1	0000	0414	0792	1139	1461	1761	2041	2304	2553	2788	1	2.2	2.1
2	3010	3222	3424	3617	3802	3979	4150	4314	4472	4624	2	4.4	4.2
3	4771	4914	5051	5185	5315	5441	5563	5682	5798	5911	3	6.6	6.3
4	6021	6128	6232	6335	6435	6532	6628	6721	6812	6902	4	8.8	8.4
5	6990	7076	7160	7243	7324	7404	7482	7559	7634	7709	5	11.0	10.5
6	7782	7853	7924	7993	8062	8129	8195	8261	8325	8388	6	13.2	12.6
7	8451	8513	8573	8633	8692	8751	8808	8865	8921	8976	7	15.4	14.7
8	9031	9085	9138	9191	9243	9294	9345	9395	9445	9494	8	17.6	16.8
9	9542	9590	9638	9685	9731	9777	9823	9868	9912	9956	9	19.8	18.9
10	0000	0043	0086	0128	0170	0212	0253	0294	0334	0374		20	19
11	0414	0453	0492	0531	0569	0607	0645	0682	0719	0755	1	2.0	1.9
12	0792	0828	0864	0899	0934	0969	1004	1038	1072	1106	2	4.0	3.8
13	1139	1173	1206	1239	1271	1303	1335	1367	1399	1430	3	6.0	5.7
14	1461	1492	1523	1553	1584	1614	1644	1673	1703	1732	4	8.0	7.6
15	1761	1790	1818	1847	1875	1903	1931	1959	1987	2014	5	10.0	9.5
16	2041	2068	2095	2122	2148	2175	2201	2227	2253	2279	6	12.0	11.4
17	2304	2330	2355	2380	2405	2430	2455	2480	2504	2529	7	14.0	13.3
18	2553	2577	2601	2625	2648	2672	2695	2718	2742	2765	8	16.0	15.2
19	2788	2810	2833	2856	2878	2900	2923	2945	2967	2989	9	18.0	17.1
20	3010	3032	3054	3075	3096	3118	3139	3160	3181	3201		18	17
21	3222	3243	3263	3284	3304	3324	3345	3365	3385	3404	1	1.8	1.7
22	3424	3444	3464	3483	3502	3522	3541	3560	3579	3598	2	3.6	3.4
23	3617	3636	3655	3674	3692	3711	3729	3747	3766	3784	3	5.4	5.1
24	3802	3820	3838	3856	3874	3892	3909	3927	3945	3962	4	7.2	6.8
25	3979	3997	4014	4031	4048	4065	4082	4099	4116	4133	5	9.0	8.5
26	4150	4166	4183	4200	4216	4232	4249	4265	4281	4298	6	10.8	10.2
27	4314	4330	4346	4362	4378	4393	4409	4425	4440	4456	7	12.6	11.9
28	4472	4487	4502	4518	4533	4548	4564	4579	4594	4609	8	14.4	13.6
29	4624	4639	4654	4669	4683	4698	4713	4728	4742	4757	9	16.2	15.3
30	4771	4786	4800	4814	4829	4843	4857	4871	4886	4900		16	15
31	4914	4928	4942	4955	4969	4983	4997	5011	5024	5038	1	1.6	1.5
32	5051	5065	5079	5092	5105	5119	5132	5145	5159	5172	2	3.2	3.0
33	5185	5198	5211	5224	5237	5250	5263	5276	5289	5302	3	4.8	4.5
34	5315	5328	5340	5353	5366	5378	5391	5403	5416	5428	4	6.4	6.0
35	5441	5453	5465	5478	5490	5502	5514	5527	5539	5551	5	8.0	7.5
36	5563	5575	5587	5599	5611	5623	5635	5647	5658	5670	6	9.6	9.0
37	5682	5694	5705	5717	5729	5740	5752	5763	5775	5786	7	11.2	10.5
38	5798	5809	5821	5832	5843	5855	5866	5877	5888	5900	8	12.8	12.0
39	5911	5922	5933	5944	5955	5966	5977	5988	5999	6010	9	14.4	13.5
40	6021	6031	6042	6053	6064	6075	6085	6096	6107	6117		14	13
41	6128	6138	6149	6160	6170	6180	6191	6201	6212	6222	1	1.4	1.3
42	6232	6243	6253	6263	6274	6284	6294	6304	6314	6325	2	2.8	2.6
43	6335	6345	6355	6365	6375	6385	6395	6405	6415	6425	3	4.2	3.9
44	6435	6444	6454	6464	6474	6484	6493	6503	6513	6522	4	5.6	5.2
45	6532	6542	6551	6561	6571	6580	6590	6599	6609	6618	5	7.0	6.5
46	6628	6637	6646	6656	6665	6675	6684	6693	6702	6712	6	8.4	7.8
47	6721	6730	6739	6749	6758	6767	6776	6785	6794	6803	7	9.8	9.1
48	6812	6821	6830	6839	6848	6857	6866	6875	6884	6893	8	11.2	10.4
49	6902	6911	6920	6928	6937	6946	6955	6964	6972	6981	9	12.6	11.7
50	6990	6998	7007	7016	7024	7033	7042	7050	7059	7067		12	11
											1	1.2	1.1
											2	2.4	2.2
											3	3.6	3.3
											4	4.8	4.4
											5	6.0	5.5
											6	7.2	6.6
											7	8.4	7.7
											8	9.6	8.8
											9	10.8	9.9
												9	8
											1	0.9	0.8
											2	1.8	1.6
											3	2.7	2.4
											4	3.6	3.2
											5	4.5	4.0
											6	5.4	4.8
											7	6.3	5.6
											8	7.2	6.4
											9	8.1	7.2
N.	0	1	2	3	4	5	6	7	8	9			

N.	0	1	2	3	4	5	6	7	8	9	P.P.
50	6990	6998	7007	7016	7024	7033	7042	7050	7059	7067	
51	7076	7084	7093	7101	7110	7118	7126	7135	7143	7152	9
52	7160	7168	7177	7185	7193	7202	7210	7218	7226	7235	1 0.9
53	7243	7251	7259	7267	7275	7284	7292	7300	7308	7316	2 1.8
											3 2.7
54	7324	7332	7340	7348	7356	7364	7372	7380	7388	7396	4 3.6
55	7404	7412	7419	7427	7435	7443	7451	7459	7466	7474	5 4.5
56	7482	7490	7497	7505	7513	7520	7528	7536	7543	7551	6 5.4
											7 6.3
57	7559	7566	7574	7582	7589	7597	7604	7612	7619	7627	8 7.2
58	7634	7642	7649	7657	7664	7672	7679	7686	7694	7701	9 8.1
59	7709	7716	7723	7731	7738	7745	7752	7760	7767	7774	
60	7782	7789	7796	7803	7810	7818	7825	7832	7839	7846	8
61	7853	7860	7868	7875	7882	7889	7896	7903	7910	7917	1 0.8
62	7924	7931	7938	7945	7952	7959	7966	7973	7980	7987	2 1.6
63	7993	8000	8007	8014	8021	8028	8035	8041	8048	8055	3 2.4
											4 3.2
64	8062	8069	8075	8082	8089	8096	8102	8109	8116	8122	5 4.0
65	8129	8136	8142	8149	8156	8162	8169	8176	8182	8189	6 4.8
66	8195	8202	8209	8215	8222	8228	8235	8241	8248	8254	7 5.6
											8 6.4
67	8261	8267	8274	8280	8287	8293	8299	8306	8312	8319	9 7.2
68	8325	8331	8338	8344	8351	8357	8363	8370	8376	8382	
69	8388	8395	8401	8407	8414	8420	8426	8432	8439	8445	7
											1 0.7
70	8451	8457	8463	8470	8476	8482	8488	8494	8500	8506	2 1.4
71	8513	8519	8525	8531	8537	8543	8549	8555	8561	8567	3 2.1
72	8573	8579	8585	8591	8597	8603	8609	8615	8621	8627	4 2.8
73	8633	8639	8645	8651	8657	8663	8669	8675	8681	8686	5 3.5
											6 4.2
74	8692	8698	8704	8710	8716	8722	8727	8733	8739	8745	7 4.9
75	8751	8756	8762	8768	8774	8779	8785	8791	8797	8802	8 5.6
76	8808	8814	8820	8825	8831	8837	8842	8848	8854	8859	9 6.3
											6
77	8865	8871	8876	8882	8887	8893	8899	8904	8910	8915	1 0.6
78	8921	8927	8932	8938	8943	8949	8954	8960	8965	8971	2 1.2
79	8976	8982	8987	8993	8998	9004	9009	9015	9020	9025	3 1.8
											4 2.4
80	9031	9036	9042	9047	9053	9058	9063	9069	9074	9079	5 3.0
81	9085	9090	9096	9101	9106	9112	9117	9122	9128	9133	6 3.6
82	9138	9143	9149	9154	9159	9165	9170	9175	9180	9186	7 4.2
83	9191	9196	9201	9206	9212	9217	9222	9227	9232	9238	8 4.8
											9 5.4
84	9243	9248	9253	9258	9263	9269	9274	9279	9284	9289	
85	9294	9299	9304	9309	9315	9320	9325	9330	9335	9340	5
86	9345	9350	9355	9360	9365	9370	9375	9380	9385	9390	1 0.5
											2 1.0
87	9395	9400	9405	9410	9415	9420	9425	9430	9435	9440	3 1.5
88	9445	9450	9455	9460	9465	9470	9474	9479	9484	9489	4 2.0
89	9494	9499	9504	9509	9513	9518	9523	9528	9533	9538	5 2.5
											6 3.0
90	9542	9547	9552	9557	9562	9566	9571	9576	9581	9586	7 3.5
91	9590	9595	9600	9605	9609	9614	9619	9624	9628	9633	8 4.0
92	9638	9643	9647	9652	9657	9661	9666	9671	9675	9680	9 4.5
93	9685	9689	9694	9699	9703	9708	9713	9717	9722	9727	
											4
94	9731	9736	9741	9745	9750	9754	9759	9763	9768	9773	1 0.4
95	9777	9782	9786	9791	9795	9800	9805	9809	9814	9818	2 0.8
96	9823	9827	9832	9836	9841	9845	9850	9854	9859	9863	3 1.2
											4 1.6
97	9868	9872	9877	9881	9886	9890	9894	9899	9903	9908	5 2.0
98	9912	9917	9921	9926	9930	9934	9939	9943	9948	9952	6 2.4
99	9956	9961	9965	9969	9974	9978	9983	9987	9991	9996	7 2.8
											8 3.2
100	0000	0004	0009	0013	0017	0022	0026	0030	0035	0039	9 3.6
N.	0	1	2	3	4	5	6	7	8	9	

N.	0	1	2	3	4	5	6	7	8	9	P.P.
100	0000	0004	0009	0013	0017	0022	0026	0030	0035	0039	
101	0043	0048	0052	0056	0060	0065	0069	0073	0077	0082	
102	0086	0090	0095	0099	0103	0107	0111	0116	0120	0124	
103	0128	0133	0137	0141	0145	0149	0154	0158	0162	0166	
104	0170	0175	0179	0183	0187	0191	0195	0199	0204	0208	
105	0212	0216	0220	0224	0228	0233	0237	0241	0245	0249	
106	0253	0257	0261	0265	0269	0273	0278	0282	0286	0290	
107	0294	0298	0302	0306	0310	0314	0318	0322	0326	0330	
108	0334	0338	0342	0346	0350	0354	0358	0362	0366	0370	
109	0374	0378	0382	0386	0390	0394	0398	0402	0406	0410	
110	0414	0418	0422	0426	0430	0434	0438	0441	0445	0449	
111	0453	0457	0461	0465	0469	0473	0477	0481	0484	0488	
112	0492	0496	0500	0504	0508	0512	0515	0519	0523	0527	
113	0531	0535	0538	0542	0546	0550	0554	0558	0561	0565	
114	0569	0573	0577	0580	0584	0588	0592	0596	0599	0603	
115	0607	0611	0615	0618	0622	0626	0630	0633	0637	0641	
116	0645	0648	0652	0656	0660	0663	0667	0671	0674	0678	
117	0682	0686	0689	0693	0697	0700	0704	0708	0711	0715	
118	0719	0722	0726	0730	0734	0737	0741	0745	0748	0752	
119	0755	0759	0763	0766	0770	0774	0777	0781	0785	0788	
120	0792	0795	0799	0803	0806	0810	0813	0817	0821	0824	
121	0828	0831	0835	0839	0842	0846	0849	0853	0856	0860	
122	0864	0867	0871	0874	0878	0881	0885	0888	0892	0896	
123	0899	0903	0906	0910	0913	0917	0920	0924	0927	0931	
124	0934	0938	0941	0945	0948	0952	0955	0959	0962	0966	
125	0969	0973	0976	0980	0983	0986	0990	0993	0997	1000	
126	1004	1007	1011	1014	1017	1021	1024	1028	1031	1035	
127	1038	1041	1045	1048	1052	1055	1059	1062	1065	1069	
128	1072	1075	1079	1082	1086	1089	1093	1096	1099	1103	
129	1106	1109	1113	1116	1119	1123	1126	1129	1133	1136	
130	1139	1143	1146	1149	1153	1156	1159	1163	1166	1169	
131	1173	1176	1179	1183	1186	1189	1193	1196	1199	1202	
132	1206	1209	1212	1216	1219	1222	1225	1229	1232	1235	
133	1239	1242	1245	1248	1252	1255	1258	1261	1265	1268	
134	1271	1274	1278	1281	1284	1287	1290	1294	1297	1300	
135	1303	1307	1310	1313	1316	1319	1323	1326	1329	1332	
136	1335	1339	1342	1345	1348	1351	1355	1358	1361	1364	
137	1367	1370	1374	1377	1380	1383	1386	1389	1392	1396	
138	1399	1402	1405	1408	1411	1414	1418	1421	1424	1427	
139	1430	1433	1436	1440	1443	1446	1449	1452	1455	1458	
140	1461	1464	1467	1471	1474	1477	1480	1483	1486	1489	
141	1492	1495	1498	1501	1504	1508	1511	1514	1517	1520	
142	1523	1526	1529	1532	1535	1538	1541	1544	1547	1550	
143	1553	1556	1559	1562	1565	1569	1572	1575	1578	1581	
144	1584	1587	1590	1593	1596	1599	1602	1605	1608	1611	
145	1614	1617	1620	1623	1626	1629	1632	1635	1638	1641	
146	1644	1647	1649	1652	1655	1658	1661	1664	1667	1670	
147	1673	1676	1679	1682	1685	1688	1691	1694	1697	1700	
148	1703	1706	1708	1711	1714	1717	1720	1723	1726	1729	
149	1732	1735	1738	1741	1744	1746	1749	1752	1755	1758	
150	1761	1764	1767	1770	1772	1775	1778	1781	1784	1787	
N.	0	1	2	3	4	5	6	7	8	9	

5
1 0.5
2 1.0
3 1.5
4 2.0
5 2.5
6 3.0
7 3.5
8 4.0
9 4.5

4
1 0.4
2 0.8
3 1.2
4 1.6
5 2.0
6 2.4
7 2.8
8 3.2
9 3.6

3
1 0.3
2 0.6
3 0.9
4 1.2
5 1.5
6 1.8
7 2.1
8 2.4
9 2.7

2
1 0.2
2 0.4
3 0.6
4 0.8
5 1.0
6 1.2
7 1.4
8 1.6
9 1.8

N.	0	1	2	3	4	5	6	7	8	9	P.P.
150	1761	1764	1767	1770	1772	1775	1778	1781	1784	1787	
151	1790	1793	1796	1798	1801	1804	1807	1810	1813	1816	
152	1818	1821	1824	1827	1830	1833	1836	1838	1841	1844	
153	1847	1850	1853	1855	1858	1861	1864	1867	1870	1872	
154	1875	1878	1881	1884	1886	1889	1892	1895	1898	1901	
155	1903	1906	1909	1912	1915	1917	1920	1923	1926	1928	
156	1931	1934	1937	1940	1942	1945	1948	1951	1953	1956	
157	1959	1962	1965	1967	1970	1973	1976	1978	1981	1984	
158	1987	1989	1992	1995	1998	2000	2003	2006	2009	2011	
159	2014	2017	2019	2022	2025	2028	2030	2033	2036	2038	
160	2041	2044	2047	2049	2052	2055	2057	2060	2063	2066	
161	2068	2071	2074	2076	2079	2082	2084	2087	2090	2092	
162	2095	2098	2101	2103	2106	2109	2111	2114	2117	2119	
163	2122	2125	2127	2130	2133	2135	2138	2140	2143	2146	
164	2148	2151	2154	2156	2159	2162	2164	2167	2170	2172	
165	2175	2177	2180	2183	2185	2188	2191	2193	2196	2198	
166	2201	2204	2206	2209	2212	2214	2217	2219	2222	2225	
167	2227	2230	2232	2235	2238	2240	2243	2245	2248	2251	
168	2253	2256	2258	2261	2263	2266	2269	2271	2274	2276	
169	2279	2281	2284	2287	2289	2292	2294	2297	2299	2302	
170	2304	2307	2310	2312	2315	2317	2320	2322	2325	2327	
171	2330	2333	2335	2338	2340	2343	2345	2348	2350	2353	
172	2355	2358	2360	2363	2365	2368	2370	2373	2375	2378	
173	2380	2383	2385	2388	2390	2393	2395	2398	2400	2403	
174	2405	2408	2410	2413	2415	2418	2420	2423	2425	2428	
175	2430	2433	2435	2438	2440	2443	2445	2448	2450	2453	
176	2455	2458	2460	2463	2465	2467	2470	2472	2475	2477	
177	2480	2482	2485	2487	2490	2492	2494	2497	2499	2502	
178	2504	2507	2509	2512	2514	2516	2519	2521	2524	2526	
179	2529	2531	2533	2536	2538	2541	2543	2545	2548	2550	
180	2553	2555	2558	2560	2562	2565	2567	2570	2572	2574	
181	2577	2579	2582	2584	2586	2589	2591	2594	2596	2598	
182	2601	2603	2605	2608	2610	2613	2615	2617	2620	2622	
183	2625	2627	2629	2632	2634	2636	2639	2641	2643	2646	
184	2648	2651	2653	2655	2658	2660	2662	2665	2667	2669	
185	2672	2674	2676	2679	2681	2683	2686	2688	2690	2693	
186	2695	2697	2700	2702	2704	2707	2709	2711	2714	2716	
187	2718	2721	2723	2725	2728	2730	2732	2735	2737	2739	
188	2742	2744	2746	2749	2751	2753	2755	2758	2760	2762	
189	2765	2767	2769	2772	2774	2776	2778	2781	2783	2785	
190	2788	2790	2792	2794	2797	2799	2801	2804	2806	2808	
191	2810	2813	2815	2817	2819	2822	2824	2826	2828	2831	
192	2833	2835	2838	2840	2842	2844	2847	2849	2851	2853	
193	2856	2858	2860	2862	2865	2867	2869	2871	2874	2876	
194	2878	2880	2883	2885	2887	2889	2891	2894	2896	2898	
195	2900	2903	2905	2907	2909	2911	2914	2916	2918	2920	
196	2923	2925	2927	2929	2931	2934	2936	2938	2940	2942	
197	2945	2947	2949	2951	2953	2956	2958	2960	2962	2964	
198	2967	2969	2971	2973	2975	2978	2980	2982	2984	2986	
199	2989	2991	2993	2995	2997	2999	3002	3004	3006	3008	
200	3010	3012	3015	3017	3019	3021	3023	3025	3028	3030	
N.	0	1	2	3	4	5	6	7	8	9	

3

1	0.3
2	0.6
3	0.9
4	1.2
5	1.5
6	1.8
7	2.1
8	2.4
9	2.7

2

1	0.2
2	0.4
3	0.6
4	0.8
5	1.0
6	1.2
7	1.4
8	1.6
9	1.8

TRIGONOMETRIC FUNCTIONS

NATURAL VALUES OF TRIGONOMETRIC FUNCTIONS AND
THEIR LOGARITHMS TO THE BASE 10

De- grees	SINE		TANGENT		COTANGENT		COSINE		De- grees
	Value	log	Value	log	Value	log	Value	log	
0	.0000	-∞	.0000	-∞	∞	∞	1.0000	0.0000	90
1	.0175	8.2419	.0175	8.2419	57.230	1.7581	.9998	9.9999	89
2	.0349	8.5428	.0349	8.5431	28.636	1.4569	.9994	9.9997	88
3	.0523	8.7188	.0524	8.7194	19.081	1.2806	.9986	9.9994	87
4	.0698	8.8436	.0699	8.8446	14.301	1.1554	.9976	9.9989	86
5	.0872	8.9403	.0875	8.9420	11.430	1.0580	.9962	9.9983	85
6	.1045	9.0192	.1051	9.0216	9.5144	0.9784	.9945	9.9976	84
7	.1219	9.0859	.1228	9.0891	8.1443	0.9109	.9925	9.9968	83
8	.1392	9.1436	.1405	9.1478	7.1154	0.8522	.9903	9.9958	82
9	.1564	9.1943	.1584	9.1997	6.3138	0.8003	.9877	9.9946	81
10	.1736	9.2397	.1763	9.2463	5.6713	0.7537	.9848	9.9934	80
11	.1908	9.2806	.1944	9.2887	5.1446	0.7113	.9816	9.9919	79
12	.2079	9.3179	.2126	9.3275	4.7046	0.6725	.9781	9.9904	78
13	.2250	9.3521	.2309	9.3634	4.3315	0.6366	.9744	9.9887	77
14	.2419	9.3837	.2493	9.3968	4.0108	0.6032	.9703	9.9869	76
15	.2588	9.4130	.2679	9.4281	3.7321	0.5719	.9659	9.9849	75
16	.2756	9.4403	.2867	9.4575	3.4874	0.5425	.9613	9.9828	74
17	.2924	9.4659	.3057	9.4853	3.2709	0.5147	.9563	9.9806	73
18	.3090	9.4900	.3249	9.5118	3.0777	0.4882	.9511	9.9782	72
19	.3256	9.5126	.3443	9.5370	2.9042	0.4630	.9455	9.9757	71
20	.3420	9.5341	.3640	9.5611	2.7475	0.4389	.9397	9.9730	70
21	.3584	9.5543	.3839	9.5842	2.6051	0.4158	.9336	9.9702	69
22	.3746	9.5736	.4040	9.6064	2.4751	0.3936	.9272	9.9672	68
23	.3907	9.5919	.4245	9.6279	2.3559	0.3721	.9205	9.9640	67
24	.4067	9.6093	.4452	9.6486	2.2460	0.3514	.9135	9.9607	66
25	.4226	9.6259	.4663	9.6687	2.1445	0.3313	.9063	9.9573	65
26	.4384	9.6418	.4877	9.6882	2.0503	0.3118	.8988	9.9537	64
27	.4540	9.6570	.5095	9.7072	1.9626	0.2928	.8910	9.9499	63
28	.4695	9.6716	.5317	9.7257	1.8807	0.2743	.8829	9.9459	62
29	.4848	9.6856	.5543	9.7438	1.8040	0.2562	.8746	9.9418	61
30	.5000	9.6990	.5774	9.7614	1.7321	0.2386	.8660	9.9375	60
31	.5150	9.7118	.6009	9.7788	1.6643	0.2212	.8572	9.9331	59
32	.5299	9.7242	.6249	9.7958	1.6003	0.2042	.8480	9.9284	58
33	.5446	9.7361	.6494	9.8125	1.5399	0.1875	.8387	9.9236	57
34	.5592	9.7476	.6745	9.8290	1.4826	0.1710	.8290	9.9186	56
35	.5736	9.7586	.7002	9.8452	1.4281	0.1548	.8192	9.9134	55
36	.5878	9.7692	.7265	9.8613	1.3764	0.1387	.8090	9.9080	54
37	.6018	9.7795	.7536	9.8771	1.3270	0.1229	.7986	9.9023	53
38	.6157	9.7893	.7813	9.8928	1.2799	0.1072	.7880	9.8965	52
39	.6293	9.7989	.8098	9.9084	1.2349	0.0916	.7771	9.8905	51
40	.6428	9.8081	.8391	9.9238	1.1918	0.0762	.7660	9.8843	50
41	.6561	9.8169	.8693	9.9392	1.1504	0.0608	.7547	9.8778	49
42	.6691	9.8255	.9004	9.9544	1.1106	0.0456	.7431	9.8711	48
43	.6820	9.8338	.9325	9.9697	1.0724	0.0303	.7314	9.8641	47
44	.6947	9.8418	.9657	9.9848	1.0355	0.0152	.7193	9.8569	46
45	.7071	9.8495	1.0000	0.0000	1.0000	0.0000	.7071	9.8495	45
	Value	log	Value	log	Value	log	Value	log	
	COSINE		COTANGENT		TANGENT		SINE		

RADIAN EQUIVALENTS OF DEGREE MEASURE 7

Deg.	Rad.	Deg.	Rad.	Deg.	Rad.	Deg.	Rad.	Deg.	Rad.	Deg.	Rad.
1	0.017	61	1.065	121	2.112	181	3.159	241	4.206	301	5.253
2	0.035	62	1.082	122	2.129	182	3.177	242	4.223	302	5.271
3	0.052	63	1.100	123	2.147	183	3.194	243	4.241	303	5.288
4	0.070	64	1.117	124	2.164	184	3.211	244	4.258	304	5.306
5	0.087	65	1.134	125	2.182	185	3.229	245	4.276	305	5.323
6	0.105	66	1.152	126	2.199	186	3.246	246	4.293	306	5.341
7	0.122	67	1.169	127	2.217	187	3.264	247	4.311	307	5.358
8	0.140	68	1.187	128	2.234	188	3.281	248	4.328	308	5.375
9	0.157	69	1.204	129	2.251	189	3.299	249	4.346	309	5.393
10	0.175	70	1.222	130	2.269	190	3.316	250	4.363	310	5.410
11	0.192	71	1.239	131	2.286	191	3.334	251	4.380	311	5.428
12	0.209	72	1.257	132	2.304	192	3.351	252	4.398	312	5.445
13	0.227	73	1.274	133	2.321	193	3.369	253	4.415	313	5.463
14	0.244	74	1.292	134	2.339	194	3.386	254	4.433	314	5.480
15	0.262	75	1.309	135	2.356	195	3.403	255	4.450	315	5.498
16	0.279	76	1.326	136	2.374	196	3.421	256	4.468	316	5.515
17	0.297	77	1.344	137	2.391	197	3.438	257	4.485	317	5.533
18	0.314	78	1.361	138	2.409	198	3.456	258	4.503	318	5.550
19	0.332	79	1.379	139	2.426	199	3.473	259	4.520	319	5.567
20	0.349	80	1.396	140	2.443	200	3.491	260	4.538	320	5.585
21	0.367	81	1.414	141	2.461	201	3.508	261	4.555	321	5.602
22	0.384	82	1.431	142	2.478	202	3.526	262	4.572	322	5.620
23	0.401	83	1.449	143	2.496	203	3.543	263	4.590	323	5.637
24	0.419	84	1.466	144	2.513	204	3.560	264	4.607	324	5.655
25	0.436	85	1.484	145	2.531	205	3.578	265	4.625	325	5.672
26	0.454	86	1.501	146	2.548	206	3.595	266	4.642	326	5.690
27	0.471	87	1.518	147	2.566	207	3.613	267	4.660	327	5.707
28	0.489	88	1.536	148	2.583	208	3.630	268	4.677	328	5.724
29	0.506	89	1.553	149	2.601	209	3.648	269	4.695	329	5.742
30	0.524	90	1.571	150	2.618	210	3.665	270	4.712	330	5.760
31	0.541	91	1.588	151	2.635	211	3.683	271	4.730	331	5.777
32	0.558	92	1.606	152	2.653	212	3.700	272	4.747	332	5.794
33	0.576	93	1.623	153	2.670	213	3.717	273	4.765	333	5.812
34	0.593	94	1.641	154	2.688	214	3.735	274	4.782	334	5.829
35	0.611	95	1.658	155	2.705	215	3.752	275	4.800	335	5.847
36	0.628	96	1.676	156	2.723	216	3.770	276	4.817	336	5.864
37	0.646	97	1.693	157	2.740	217	3.787	277	4.835	337	5.882
38	0.663	98	1.710	158	2.758	218	3.805	278	4.852	338	5.899
39	0.681	99	1.728	159	2.775	219	3.822	279	4.869	339	5.916
40	0.698	100	1.745	160	2.793	220	3.840	280	4.887	340	5.934
41	0.715	101	1.763	161	2.810	221	3.857	281	4.904	341	5.951
42	0.733	102	1.780	162	2.827	222	3.874	282	4.922	342	5.969
43	0.750	103	1.798	163	2.845	223	3.892	283	4.939	343	5.986
44	0.768	104	1.815	164	2.862	224	3.909	284	4.957	344	6.004
45	0.785	105	1.833	165	2.880	225	3.927	285	4.974	345	6.021
46	0.803	106	1.850	166	2.897	226	3.944	286	4.992	346	6.039
47	0.820	107	1.868	167	2.915	227	3.962	287	5.009	347	6.056
48	0.838	108	1.885	168	2.932	228	3.979	288	5.026	348	6.073
49	0.855	109	1.902	169	2.950	229	3.997	289	5.044	349	6.091
50	0.873	110	1.920	170	2.967	230	4.014	290	5.061	350	6.108
51	0.890	111	1.937	171	2.985	231	4.031	291	5.079	351	6.126
52	0.908	112	1.955	172	3.002	232	4.049	292	5.096	352	6.143
53	0.925	113	1.972	173	3.019	233	4.066	293	5.114	353	6.161
54	0.942	114	1.990	174	3.037	234	4.084	294	5.131	354	6.178
55	0.960	115	2.007	175	3.054	235	4.101	295	5.149	355	6.196
56	0.977	116	2.025	176	3.072	236	4.119	296	5.166	356	6.213
57	0.995	117	2.042	177	3.089	237	4.136	297	5.184	357	6.231
58	1.012	118	2.059	178	3.107	238	4.154	298	5.201	358	6.248
59	1.030	119	2.077	179	3.124	239	4.171	299	5.218	359	6.265
60	1.047	120	2.094	180	3.142	240	4.189	300	5.236	360	6.283

NATURAL VALUES OF TRIGONOMETRIC FUNCTIONS FOR ANGLES IN RADIAN MEASURE

Rad.	Deg.	Min.	sin	tan	cot	cos
0.05	2	51.9	.050	.050	19.98	.999
0.10	5	43.8	.100	.100	9.995	.995
0.15	8	35.7	.149	.151	6.616	.989
0.20	11	27.5	.199	.203	4.933	.980
0.25	14	19.4	.247	.255	3.916	.969
0.30	17	11.3	.296	.309	3.233	.955
0.35	20	3.2	.343	.365	2.739	.939
0.40	22	55.1	.389	.423	2.365	.921
0.45	25	47.0	.435	.483	2.070	.900
0.50	28	38.9	.479	.546	1.830	.878
0.55	31	30.8	.523	.613	1.631	.853
0.60	34	22.6	.565	.684	1.462	.825
0.65	37	14.5	.605	.760	1.315	.796
0.70	40	6.4	.644	.842	1.187	.765
0.75	42	58.3	.682	.932	1.073	.732
0.80	45	50.2	.717	1.030	.971	.697
0.85	48	42.1	.751	1.138	.878	.660
0.90	51	34.0	.783	1.260	.794	.622
0.95	54	25.9	.813	1.398	.715	.582
1.00	57	17.7	.841	1.557	.642	.540
1.05	60	9.6	.867	1.743	.574	.498
1.10	63	1.5	.891	1.964	.509	.454
1.15	65	53.4	.913	2.234	.447	.408
1.20	68	45.3	.932	2.572	.389	.362
1.25	71	37.2	.949	3.010	.332	.315
1.30	74	29.1	.964	3.602	.278	.268
1.35	77	20.9	.976	4.454	.224	.219
1.40	80	12.8	.985	5.797	.172	.170
1.45	83	4.7	.993	8.237	.121	.121
1.50	85	56.6	.997	14.10	.071	.071
1.55	88	48.5	1.000	48.08	.021	.021
1.60	91	40.4	1.000	-34.27	-.029	-.029
1.65	94	32.2	.997	-12.62	-.079	-.079
1.70	97	24.1	.992	-7.700	-.130	-.129
1.75	100	16.0	.984	-5.521	-.181	-.178
1.80	103	7.9	.974	-4.286	-.233	-.227
1.85	105	59.8	.961	-3.488	-.287	-.276
1.90	108	51.7	.946	-2.927	-.342	-.323
1.95	111	43.6	.929	-2.510	-.398	-.370
2.00	114	35.4	.909	-2.185	-.458	-.416
2.05	117	27.3	.887	-1.924	-.520	-.461
2.10	120	19.2	.863	-1.710	-.585	-.505
2.15	123	11.1	.837	-1.529	-.654	-.547
2.20	126	3.0	.808	-1.374	-.728	-.588
2.25	128	54.9	.778	-1.239	-.807	-.628
2.30	131	46.8	.746	-1.119	-.894	-.666
2.35	134	38.6	.711	-1.012	-.988	-.703
2.40	137	30.5	.675	-.916	-1.092	-.737
2.45	140	22.4	.638	-.828	-1.208	-.770
2.50	143	14.3	.598	-.747	-1.339	-.801
2.55	146	6.2	.558	-.672	-1.488	-.830
2.60	148	58.1	.515	-.602	-1.662	-.857
2.65	151	50.0	.472	-.535	-1.868	-.881
2.70	154	41.9	.427	-.473	-2.115	-.904
2.75	157	33.7	.382	-.413	-2.423	-.924
2.80	160	25.6	.335	-.356	-2.813	-.942
2.85	163	17.5	.287	-.300	-3.331	-.958
2.90	166	9.4	.239	-.246	-4.059	-.971
2.95	169	1.3	.190	-.194	-5.155	-.982
3.00	171	53.1	.141	-.143	-7.015	-.990

SQUARES AND CUBES; SQUARE ROOTS AND CUBE ROOTS

No.	Square	Cube	Square Root	Cube Root	No.	Square	Cube	Square Root	Cube Root
1	1	1	1.000	1.000	51	2,601	132,651	7.141	3.708
2	4	8	1.414	1.259	52	2,704	140,608	7.211	3.732
3	9	27	1.732	1.442	53	2,809	148,877	7.280	3.756
4	16	64	2.000	1.587	54	2,916	157,464	7.348	3.779
5	25	125	2.236	1.709	55	3,025	166,375	7.416	3.802
6	36	216	2.449	1.817	56	3,136	175,616	7.483	3.825
7	49	343	2.645	1.912	57	3,249	185,193	7.549	3.848
8	64	512	2.828	2.000	58	3,364	195,112	7.615	3.870
9	81	729	3.000	2.080	59	3,481	205,379	7.681	3.892
10	100	1,000	3.162	2.154	60	3,600	216,000	7.745	3.914
11	121	1,331	3.316	2.223	61	3,721	226,981	7.810	3.936
12	144	1,728	3.464	2.289	62	3,844	238,328	7.874	3.957
13	169	2,197	3.605	2.351	63	3,969	250,047	7.937	3.979
14	196	2,744	3.741	2.410	64	4,096	262,144	8.000	4.000
15	225	3,375	3.872	2.466	65	4,225	274,625	8.062	4.020
16	256	4,096	4.000	2.519	66	4,356	287,496	8.124	4.041
17	289	4,913	4.123	2.571	67	4,489	300,763	8.185	4.061
18	324	5,832	4.242	2.620	68	4,624	314,432	8.246	4.081
19	361	6,859	4.358	2.668	69	4,761	328,509	8.306	4.101
20	400	8,000	4.472	2.714	70	4,900	343,000	8.366	4.121
21	441	9,261	4.582	2.758	71	5,041	357,911	8.426	4.140
22	484	10,648	4.690	2.802	72	5,184	373,248	8.485	4.160
23	529	12,167	4.795	2.843	73	5,329	389,017	8.544	4.179
24	576	13,824	4.898	2.884	74	5,476	405,224	8.602	4.198
25	625	15,625	5.000	2.924	75	5,625	421,875	8.660	4.217
26	676	17,576	5.099	2.962	76	5,776	438,976	8.717	4.235
27	729	19,683	5.196	3.000	77	5,929	456,533	8.774	4.254
28	784	21,952	5.291	3.036	78	6,084	474,552	8.831	4.272
29	841	24,389	5.385	3.072	79	6,241	493,039	8.888	4.290
30	900	27,000	5.477	3.107	80	6,400	512,000	8.944	4.308
31	961	29,791	5.567	3.141	81	6,561	531,441	9.000	4.326
32	1,024	32,768	5.656	3.174	82	6,724	551,368	9.055	4.344
33	1,089	35,937	5.744	3.207	83	6,889	571,787	9.110	4.362
34	1,156	39,304	5.830	3.239	84	7,056	592,704	9.165	4.379
35	1,225	42,875	5.916	3.271	85	7,225	614,125	9.219	4.396
36	1,296	46,656	6.000	3.301	86	7,396	636,056	9.273	4.414
37	1,369	50,653	6.082	3.332	87	7,569	658,503	9.327	4.431
38	1,444	54,872	6.164	3.361	88	7,744	681,472	9.380	4.447
39	1,521	59,319	6.244	3.391	89	7,921	704,969	9.433	4.464
40	1,600	64,000	6.324	3.419	90	8,100	729,000	9.486	4.481
41	1,681	68,921	6.403	3.448	91	8,281	753,571	9.539	4.497
42	1,764	74,088	6.480	3.476	92	8,464	778,688	9.591	4.514
43	1,849	79,507	6.557	3.503	93	8,649	804,357	9.643	4.530
44	1,936	85,184	6.633	3.530	94	8,836	830,584	9.695	4.546
45	2,025	91,125	6.708	3.556	95	9,025	857,375	9.746	4.562
46	2,116	97,336	6.782	3.583	96	9,216	884,736	9.797	4.578
47	2,209	103,823	6.855	3.608	97	9,409	912,673	9.848	4.594
48	2,304	110,592	6.928	3.634	98	9,604	941,192	9.899	4.610
49	2,401	117,649	7.000	3.659	99	9,801	970,299	9.949	4.626
50	2,500	125,000	7.071	3.684	100	10,000	1,000,000	10.000	4.641

N.	0	1	2	3	4	5	6	7	8	9
0	0.000	1.000	1.414	1.732	2.000	2.236	2.449	2.645	2.828	3.000
1	3.162	3.316	3.464	3.605	3.741	3.873	4.000	4.123	4.242	4.358
2	4.472	4.582	4.690	4.795	4.898	5.000	5.099	5.196	5.291	5.385
3	5.477	5.567	5.656	5.744	5.830	5.916	6.000	6.082	6.164	6.244
4	6.324	6.403	6.480	6.557	6.633	6.708	6.782	6.855	6.928	7.000
5	7.071	7.141	7.211	7.280	7.348	7.416	7.483	7.549	7.615	7.681
6	7.745	7.810	7.874	7.937	8.000	8.062	8.124	8.185	8.246	8.306
7	8.366	8.426	8.485	8.544	8.602	8.660	8.717	8.774	8.831	8.888
8	8.944	9.000	9.055	9.110	9.165	9.219	9.273	9.327	9.380	9.433
9	9.486	9.539	9.591	9.643	9.695	9.746	9.797	9.848	9.899	9.949
10	10.000	10.049	10.099	10.148	10.198	10.247	10.295	10.344	10.392	10.440
11	10.488	10.535	10.583	10.630	10.677	10.723	10.770	10.816	10.862	10.908
12	10.954	11.000	11.045	11.090	11.135	11.180	11.225	11.269	11.313	11.357
13	11.401	11.445	11.489	11.532	11.575	11.618	11.661	11.704	11.747	11.789
14	11.832	11.874	11.916	11.958	12.000	12.041	12.083	12.124	12.165	12.206
15	12.247	12.288	12.329	12.369	12.409	12.449	12.490	12.530	12.569	12.609
16	12.649	12.688	12.727	12.767	12.806	12.845	12.884	12.922	12.961	13.000
17	13.038	13.076	13.114	13.152	13.190	13.228	13.266	13.304	13.341	13.379
18	13.416	13.453	13.490	13.527	13.564	13.601	13.638	13.674	13.711	13.747
19	13.784	13.820	13.856	13.892	13.928	13.964	14.000	14.035	14.071	14.106
20	14.142	14.177	14.212	14.247	14.282	14.317	14.352	14.387	14.422	14.456
21	14.491	14.525	14.560	14.594	14.628	14.662	14.696	14.730	14.764	14.798
22	14.832	14.866	14.899	14.933	14.966	15.000	15.033	15.066	15.099	15.132
23	15.165	15.198	15.231	15.264	15.297	15.329	15.362	15.394	15.427	15.459
24	15.491	15.524	15.556	15.588	15.620	15.652	15.684	15.716	15.748	15.779
25	15.811	15.843	15.874	15.906	15.937	15.968	16.000	16.031	16.062	16.093
26	16.124	16.155	16.186	16.217	16.248	16.278	16.309	16.340	16.370	16.401
27	16.431	16.462	16.492	16.522	16.552	16.583	16.613	16.643	16.673	16.703
28	16.733	16.763	16.792	16.822	16.852	16.881	16.911	16.941	16.970	17.000
29	17.029	17.058	17.088	17.117	17.146	17.175	17.204	17.233	17.262	17.291
30	17.320	17.349	17.378	17.406	17.435	17.464	17.492	17.521	17.549	17.578
31	17.606	17.635	17.663	17.691	17.720	17.748	17.776	17.804	17.832	17.860
32	17.888	17.916	17.944	17.972	18.000	18.027	18.055	18.083	18.110	18.138
33	18.165	18.193	18.220	18.248	18.275	18.303	18.330	18.357	18.384	18.412
34	18.439	18.466	18.493	18.520	18.547	18.574	18.601	18.627	18.654	18.681
35	18.708	18.735	18.761	18.788	18.814	18.841	18.868	18.894	18.920	18.947
36	18.973	19.000	19.026	19.052	19.078	19.105	19.131	19.157	19.183	19.209
37	19.235	19.261	19.287	19.313	19.339	19.364	19.390	19.416	19.442	19.467
38	19.493	19.519	19.544	19.570	19.595	19.621	19.646	19.672	19.697	19.723
39	19.748	19.773	19.799	19.824	19.849	19.874	19.899	19.924	19.949	19.975
40	20.000	20.025	20.049	20.074	20.099	20.124	20.149	20.174	20.199	20.223
41	20.248	20.273	20.297	20.322	20.347	20.371	20.396	20.420	20.445	20.469
42	20.493	20.518	20.542	20.567	20.591	20.615	20.639	20.664	20.688	20.712
43	20.736	20.760	20.784	20.808	20.832	20.856	20.880	20.904	20.928	20.952
44	20.976	21.000	21.023	21.047	21.071	21.095	21.118	21.142	21.166	21.189
45	21.213	21.236	21.260	21.283	21.307	21.330	21.354	21.377	21.400	21.424
46	21.447	21.470	21.494	21.517	21.540	21.563	21.587	21.610	21.633	21.656
47	21.679	21.702	21.725	21.748	21.771	21.794	21.817	21.840	21.863	21.886
48	21.908	21.931	21.954	21.977	22.000	22.022	22.045	22.068	22.090	22.113
49	22.135	22.158	22.181	22.203	22.226	22.248	22.271	22.293	22.315	22.338
50	22.360	22.383	22.405	22.427	22.449	22.472	22.494	22.516	22.538	22.561
N.	0	1	2	3	4	5	6	7	8	9

SQUARE ROOTS

11

N.	0	1	2	3	4	5	6	7	8	9
50	22.360	22.383	22.405	22.427	22.449	22.472	22.494	22.516	22.538	22.561
51	22.583	22.605	22.627	22.649	22.671	22.693	22.715	22.737	22.759	22.781
52	22.803	22.825	22.847	22.869	22.891	22.912	22.934	22.956	22.978	23.000
53	23.021	23.043	23.065	23.086	23.108	23.130	23.151	23.173	23.194	23.216
54	23.237	23.259	23.280	23.302	23.323	23.345	23.366	23.388	23.409	23.430
55	23.452	23.473	23.494	23.516	23.537	23.558	23.579	23.600	23.622	23.643
56	23.664	23.685	23.706	23.727	23.748	23.769	23.790	23.811	23.832	23.853
57	23.874	23.895	23.916	23.937	23.958	23.979	24.000	24.020	24.041	24.062
58	24.083	24.103	24.124	24.145	24.166	24.186	24.207	24.228	24.248	24.269
59	24.289	24.310	24.331	24.351	24.372	24.392	24.413	24.433	24.454	24.474
60	24.494	24.515	24.535	24.556	24.576	24.596	24.617	24.637	24.657	24.677
61	24.698	24.718	24.738	24.758	24.779	24.799	24.819	24.839	24.859	24.879
62	24.899	24.919	24.939	24.960	24.980	25.000	25.020	25.040	25.060	25.079
63	25.099	25.119	25.139	25.159	25.179	25.199	25.219	25.238	25.258	25.278
64	25.298	25.318	25.337	25.357	25.377	25.396	25.416	25.436	25.455	25.475
65	25.495	25.514	25.534	25.553	25.573	25.593	25.612	25.632	25.651	25.671
66	25.690	25.709	25.729	25.748	25.768	25.787	25.807	25.826	25.845	25.865
67	25.884	25.903	25.923	25.942	25.961	25.980	26.000	26.019	26.038	26.057
68	26.076	26.096	26.115	26.134	26.153	26.172	26.191	26.210	26.229	26.248
69	26.267	26.286	26.305	26.324	26.343	26.362	26.381	26.400	26.419	26.438
70	26.457	26.476	26.495	26.514	26.533	26.551	26.570	26.589	26.608	26.627
71	26.645	26.664	26.683	26.702	26.720	26.739	26.758	26.776	26.795	26.814
72	26.832	26.851	26.870	26.888	26.907	26.925	26.944	26.962	26.981	27.000
73	27.018	27.037	27.055	27.074	27.092	27.110	27.129	27.147	27.166	27.184
74	27.202	27.221	27.239	27.258	27.276	27.294	27.313	27.331	27.349	27.367
75	27.386	27.404	27.422	27.440	27.459	27.477	27.495	27.513	27.531	27.550
76	27.568	27.586	27.604	27.622	27.640	27.658	27.676	27.694	27.712	27.730
77	27.748	27.766	27.784	27.802	27.820	27.838	27.856	27.874	27.892	27.910
78	27.928	27.946	27.964	27.982	28.000	28.017	28.035	28.053	28.071	28.089
79	28.106	28.124	28.142	28.160	28.178	28.195	28.213	28.231	28.248	28.266
80	28.284	28.301	28.319	28.337	28.354	28.372	28.390	28.407	28.425	28.442
81	28.460	28.478	28.495	28.513	28.530	28.548	28.565	28.583	28.600	28.618
82	28.635	28.653	28.670	28.688	28.705	28.722	28.740	28.757	28.775	28.792
83	28.809	28.827	28.844	28.861	28.879	28.896	28.913	28.931	28.948	28.965
84	28.982	29.000	29.017	29.034	29.051	29.068	29.086	29.103	29.120	29.137
85	29.154	29.171	29.189	29.206	29.223	29.240	29.257	29.274	29.291	29.308
86	29.325	29.342	29.359	29.376	29.393	29.410	29.427	29.444	29.461	29.478
87	29.495	29.512	29.529	29.546	29.563	29.580	29.597	29.614	29.631	29.647
88	29.664	29.681	29.698	29.715	29.732	29.748	29.765	29.782	29.799	29.816
89	29.832	29.849	29.866	29.883	29.899	29.916	29.933	29.950	29.966	29.983
90	30.000	30.016	30.033	30.050	30.066	30.083	30.099	30.116	30.133	30.149
91	30.166	30.182	30.199	30.215	30.232	30.249	30.265	30.282	30.298	30.315
92	30.331	30.348	30.364	30.380	30.397	30.413	30.430	30.446	30.463	30.479
93	30.495	30.512	30.528	30.545	30.561	30.577	30.594	30.610	30.626	30.643
94	30.659	30.675	30.692	30.708	30.724	30.740	30.757	30.773	30.789	30.805
95	30.822	30.838	30.854	30.870	30.886	30.903	30.919	30.935	30.951	30.967
96	30.983	31.000	31.016	31.032	31.048	31.064	31.080	31.096	31.112	31.128
97	31.144	31.160	31.176	31.192	31.209	31.225	31.241	31.257	31.273	31.289
98	31.305	31.320	31.336	31.352	31.368	31.384	31.400	31.416	31.432	31.448
99	31.464	31.480	31.496	31.511	31.527	31.543	31.559	31.575	31.591	31.607
100	31.622	31.638	31.654	31.670	31.685	31.701	31.717	31.733	31.749	31.764
N.	0	1	2	3	4	5	6	7	8	9

N.	0	1	2	3	4	5	6	7	8	9
0	0.000	1.000	1.259	1.442	1.587	1.709	1.817	1.912	2.000	2.080
1	2.154	2.223	2.289	2.351	2.410	2.466	2.519	2.571	2.620	2.668
2	2.714	2.758	2.802	2.843	2.884	2.924	2.962	3.000	3.036	3.072
3	3.107	3.141	3.174	3.207	3.239	3.271	3.301	3.332	3.361	3.391
4	3.419	3.448	3.476	3.503	3.530	3.556	3.583	3.608	3.634	3.659
5	3.684	3.708	3.732	3.756	3.779	3.802	3.825	3.848	3.870	3.892
6	3.914	3.936	3.957	3.979	4.000	4.020	4.041	4.061	4.081	4.101
7	4.121	4.140	4.160	4.179	4.198	4.217	4.235	4.254	4.272	4.290
8	4.308	4.326	4.344	4.362	4.379	4.396	4.414	4.431	4.447	4.464
9	4.481	4.497	4.514	4.530	4.546	4.562	4.578	4.594	4.610	4.626
10	4.641	4.657	4.672	4.687	4.702	4.717	4.732	4.747	4.762	4.776
11	4.791	4.805	4.820	4.834	4.848	4.862	4.877	4.891	4.904	4.918
12	4.932	4.946	4.959	4.973	4.986	5.000	5.013	5.026	5.039	5.052
13	5.065	5.078	5.091	5.104	5.117	5.129	5.142	5.155	5.167	5.180
14	5.192	5.204	5.217	5.229	5.241	5.253	5.265	5.277	5.289	5.301
15	5.313	5.325	5.336	5.348	5.360	5.371	5.383	5.394	5.406	5.417
16	5.428	5.440	5.451	5.462	5.473	5.484	5.495	5.506	5.517	5.528
17	5.539	5.550	5.561	5.572	5.582	5.593	5.604	5.614	5.625	5.635
18	5.646	5.656	5.667	5.677	5.687	5.698	5.708	5.718	5.728	5.738
19	5.748	5.758	5.768	5.778	5.788	5.798	5.808	5.818	5.828	5.838
20	5.848	5.857	5.867	5.877	5.886	5.896	5.905	5.915	5.925	5.934
21	5.943	5.953	5.962	5.972	5.981	5.990	6.000	6.009	6.018	6.027
22	6.036	6.045	6.055	6.064	6.073	6.082	6.091	6.100	6.109	6.118
23	6.126	6.135	6.144	6.153	6.162	6.171	6.179	6.188	6.197	6.205
24	6.214	6.223	6.231	6.240	6.248	6.257	6.265	6.274	6.282	6.291
25	6.299	6.308	6.316	6.324	6.333	6.341	6.349	6.357	6.366	6.374
26	6.382	6.390	6.398	6.407	6.415	6.423	6.431	6.439	6.447	6.455
27	6.463	6.471	6.479	6.487	6.495	6.503	6.510	6.518	6.526	6.534
28	6.542	6.549	6.557	6.565	6.573	6.580	6.588	6.596	6.603	6.611
29	6.619	6.626	6.634	6.641	6.649	6.656	6.664	6.671	6.679	6.686
30	6.694	6.701	6.709	6.716	6.724	6.731	6.738	6.746	6.753	6.760
31	6.767	6.775	6.782	6.789	6.796	6.804	6.811	6.818	6.825	6.832
32	6.839	6.847	6.854	6.861	6.868	6.875	6.882	6.889	6.896	6.903
33	6.910	6.917	6.924	6.931	6.938	6.945	6.952	6.958	6.965	6.972
34	6.979	6.986	6.993	7.000	7.006	7.013	7.020	7.027	7.033	7.040
35	7.047	7.054	7.060	7.067	7.074	7.080	7.087	7.094	7.100	7.107
36	7.113	7.120	7.126	7.133	7.140	7.146	7.153	7.159	7.166	7.172
37	7.179	7.185	7.192	7.198	7.204	7.211	7.217	7.224	7.230	7.236
38	7.243	7.249	7.255	7.262	7.268	7.274	7.281	7.287	7.293	7.299
39	7.306	7.312	7.318	7.324	7.331	7.337	7.343	7.349	7.355	7.361
40	7.368	7.374	7.380	7.386	7.392	7.398	7.404	7.410	7.416	7.422
41	7.428	7.434	7.440	7.446	7.452	7.458	7.464	7.470	7.476	7.482
42	7.488	7.494	7.500	7.506	7.512	7.518	7.524	7.530	7.536	7.542
43	7.547	7.553	7.559	7.565	7.571	7.577	7.582	7.588	7.594	7.600
44	7.605	7.611	7.617	7.623	7.628	7.634	7.640	7.646	7.651	7.657
45	7.663	7.668	7.674	7.680	7.685	7.691	7.697	7.702	7.708	7.713
46	7.719	7.725	7.730	7.736	7.741	7.747	7.752	7.758	7.763	7.769
47	7.775	7.780	7.786	7.791	7.797	7.802	7.807	7.813	7.818	7.824
48	7.829	7.835	7.840	7.846	7.851	7.856	7.862	7.867	7.873	7.878
49	7.883	7.889	7.894	7.899	7.905	7.910	7.915	7.921	7.926	7.931
50	7.937	7.942	7.947	7.952	7.958	7.963	7.968	7.973	7.979	7.984
N.	0	1	2	3	4	5	6	7	8	9

N.	0	1	2	3	4	5	6	7	8	9
50	7.937	7.942	7.947	7.952	7.958	7.963	7.968	7.973	7.979	7.984
51	7.989	7.994	8.000	8.005	8.010	8.015	8.020	8.026	8.031	8.036
52	8.041	8.046	8.051	8.056	8.062	8.067	8.072	8.077	8.082	8.087
53	8.092	8.097	8.102	8.107	8.113	8.118	8.123	8.128	8.133	8.138
54	8.143	8.148	8.153	8.158	8.163	8.168	8.173	8.178	8.183	8.188
55	8.193	8.198	8.203	8.208	8.213	8.218	8.222	8.227	8.232	8.237
56	8.242	8.247	8.252	8.257	8.262	8.267	8.271	8.276	8.281	8.286
57	8.291	8.296	8.301	8.305	8.310	8.315	8.320	8.325	8.330	8.334
58	8.339	8.344	8.349	8.353	8.358	8.363	8.368	8.373	8.377	8.382
59	8.387	8.391	8.396	8.401	8.406	8.410	8.415	8.420	8.424	8.429
60	8.434	8.439	8.443	8.448	8.453	8.457	8.462	8.466	8.471	8.476
61	8.480	8.485	8.490	8.494	8.499	8.504	8.508	8.513	8.517	8.522
62	8.527	8.531	8.536	8.540	8.545	8.549	8.554	8.559	8.563	8.568
63	8.572	8.577	8.581	8.586	8.590	8.595	8.599	8.604	8.608	8.613
64	8.617	8.622	8.626	8.631	8.635	8.640	8.644	8.649	8.653	8.657
65	8.662	8.666	8.671	8.675	8.680	8.684	8.688	8.693	8.697	8.702
66	8.706	8.711	8.715	8.719	8.724	8.728	8.732	8.737	8.741	8.745
67	8.750	8.754	8.759	8.763	8.767	8.772	8.776	8.780	8.785	8.789
68	8.793	8.798	8.802	8.806	8.810	8.815	8.819	8.823	8.828	8.832
69	8.836	8.840	8.845	8.849	8.853	8.857	8.862	8.866	8.870	8.874
70	8.879	8.883	8.887	8.891	8.895	8.900	8.904	8.908	8.912	8.916
71	8.921	8.925	8.929	8.933	8.937	8.942	8.946	8.950	8.954	8.958
72	8.962	8.966	8.971	8.975	8.979	8.983	8.987	8.991	8.995	9.000
73	9.004	9.008	9.012	9.016	9.020	9.024	9.028	9.032	9.036	9.040
74	9.045	9.049	9.053	9.057	9.061	9.065	9.069	9.073	9.077	9.081
75	9.085	9.089	9.093	9.097	9.101	9.105	9.109	9.113	9.117	9.121
76	9.125	9.129	9.133	9.137	9.141	9.145	9.149	9.153	9.157	9.161
77	9.165	9.169	9.173	9.177	9.181	9.185	9.189	9.193	9.197	9.201
78	9.205	9.209	9.213	9.217	9.220	9.224	9.228	9.232	9.236	9.240
79	9.244	9.248	9.252	9.256	9.259	9.263	9.267	9.271	9.275	9.279
80	9.283	9.286	9.290	9.294	9.298	9.302	9.306	9.310	9.313	9.317
81	9.321	9.325	9.329	9.333	9.337	9.340	9.344	9.348	9.352	9.356
82	9.359	9.363	9.367	9.371	9.375	9.378	9.382	9.386	9.390	9.394
83	9.397	9.401	9.405	9.409	9.412	9.416	9.420	9.424	9.427	9.431
84	9.435	9.439	9.442	9.446	9.450	9.454	9.457	9.461	9.465	9.469
85	9.472	9.476	9.480	9.483	9.487	9.491	9.494	9.498	9.502	9.506
86	9.509	9.513	9.517	9.520	9.524	9.528	9.531	9.535	9.539	9.542
87	9.546	9.550	9.553	9.557	9.561	9.564	9.568	9.571	9.575	9.579
88	9.582	9.586	9.590	9.593	9.597	9.600	9.604	9.608	9.611	9.615
89	9.619	9.622	9.626	9.629	9.633	9.637	9.640	9.644	9.647	9.651
90	9.654	9.658	9.662	9.665	9.669	9.672	9.676	9.679	9.683	9.687
91	9.690	9.694	9.697	9.701	9.704	9.708	9.711	9.715	9.718	9.722
92	9.725	9.729	9.732	9.736	9.740	9.743	9.747	9.750	9.754	9.757
93	9.761	9.764	9.768	9.771	9.775	9.778	9.781	9.785	9.788	9.792
94	9.795	9.799	9.802	9.806	9.809	9.813	9.816	9.820	9.823	9.827
95	9.830	9.833	9.837	9.840	9.844	9.847	9.851	9.854	9.858	9.861
96	9.864	9.868	9.871	9.875	9.878	9.881	9.885	9.888	9.892	9.895
97	9.899	9.902	9.905	9.909	9.912	9.916	9.919	9.922	9.926	9.929
98	9.932	9.936	9.939	9.943	9.946	9.949	9.953	9.956	9.959	9.963
99	9.966	9.969	9.973	9.976	9.980	9.983	9.986	9.990	9.993	9.996
100	10.000	10.003	10.006	10.010	10.013	10.016	10.020	10.023	10.026	10.030
N.	0	1	2	3	4	5	6	7	8	9

Table of Reciprocals of Numbers from 1 to 10, at Intervals of .1

N.	.0	.1	.2	.3	.4	.5	.6	.7	.8	.9
1	1.000	.909	.833	.769	.714	.667	.625	.588	.556	.526
2	.500	.476	.455	.435	.417	.400	.385	.370	.357	.345
3	.333	.323	.313	.303	.294	.286	.278	.270	.263	.256
4	.250	.244	.238	.233	.227	.222	.217	.213	.208	.204
5	.200	.196	.192	.189	.185	.182	.179	.175	.172	.169
6	.167	.164	.161	.159	.156	.154	.152	.149	.147	.145
7	.143	.141	.139	.137	.135	.133	.132	.130	.128	.127
8	.125	.123	.122	.120	.119	.118	.116	.115	.114	.112
9	.111	.110	.109	.108	.106	.105	.104	.103	.102	.101

Table of Napierian Logarithms. Base e

N.	.0	.1	.2	.3	.4	.5	.6	.7	.8	.9
1	0	.095	.182	.262	.336	.405	.470	.531	.588	.642
2	.693	.742	.788	.833	.875	.916	.956	.993	1.030	1.065
3	1.099	1.131	1.163	1.194	1.224	1.253	1.281	1.308	1.335	1.361
4	1.386	1.411	1.435	1.459	1.482	1.504	1.526	1.548	1.569	1.589
5	1.609	1.629	1.649	1.668	1.686	1.705	1.723	1.740	1.758	1.775
6	1.792	1.808	1.825	1.841	1.856	1.872	1.887	1.902	1.917	1.932
7	1.946	1.960	1.974	1.988	2.001	2.015	2.028	2.041	2.054	2.067
8	2.079	2.092	2.104	2.116	2.128	2.140	2.152	2.163	2.175	2.186
9	2.197	2.208	2.219	2.230	2.241	2.251	2.262	2.272	2.282	2.293
$\log 10 = 2.303,$ $\log 10^2 = 4.605,$ $\log 10^3 = 6.908$										

N.	0	1	2	3	4	5	6	7	8	9
1	2.303	2.398	2.485	2.565	2.639	2.708	2.773	2.833	2.890	2.944
2	2.996	3.045	3.091	3.135	3.178	3.219	3.258	3.296	3.332	3.367
3	3.401	3.434	3.466	3.497	3.526	3.555	3.584	3.611	3.638	3.664
4	3.689	3.714	3.738	3.761	3.784	3.807	3.829	3.850	3.871	3.892
5	3.912	3.932	3.951	3.970	3.989	4.007	4.025	4.043	4.060	4.078
6	4.094	4.111	4.127	4.143	4.159	4.174	4.190	4.205	4.220	4.234
7	4.249	4.263	4.277	4.290	4.304	4.317	4.331	4.344	4.357	4.369
8	4.382	4.394	4.407	4.419	4.431	4.443	4.454	4.466	4.477	4.489
9	4.500	4.511	4.522	4.533	4.543	4.554	4.564	4.575	4.585	4.595

Conversion of Logarithms

$\log_{10} N = 0.434 \log_e N$, $\log_e N = 2.30 \log_{10} N$, where N is any number.

$\log ab = \log a + \log b$. $\log \frac{a}{b} = \log a - \log b$.

$\log a^n = n \log a$.

$$\text{Hyperbolic sine of } x = \sinh x = \frac{e^x - e^{-x}}{2}.$$

$$\text{Hyperbolic cosine of } x = \cosh x = \frac{e^x + e^{-x}}{2}.$$

$$\text{Hyperbolic tangent of } x = \tanh x = \frac{e^x - e^{-x}}{e^x + e^{-x}}.$$

x	e^x	e^{-x}	$\sinh x$	$\cosh x$	$\tanh x$
0.0	1.000	1.000	.000	1.000	.000
0.1	1.105	.905	.100	1.005	.100
0.2	1.221	.819	.201	1.020	.197
0.3	1.350	.741	.305	1.045	.291
0.4	1.492	.670	.411	1.081	.380
0.5	1.649	.607	.521	1.128	.462
0.6	1.822	.549	.637	1.185	.537
0.7	2.014	.497	.759	1.255	.604
0.8	2.226	.449	.888	1.337	.664
0.9	2.460	.407	1.027	1.433	.716
1.0	2.718	.368	1.175	1.543	.762
1.1	3.004	.333	1.336	1.669	.801
1.2	3.320	.301	1.509	1.811	.834
1.3	3.669	.273	1.698	1.971	.862
1.4	4.055	.247	1.904	2.151	.885
1.5	4.482	.223	2.129	2.352	.905
1.6	4.953	.202	2.376	2.577	.922
1.7	5.474	.183	2.646	2.828	.935
1.8	6.050	.165	2.942	3.107	.947
1.9	6.686	.150	3.268	3.418	.956
2.0	7.389	.135	3.627	3.762	.964
2.1	8.166	.122	4.022	4.144	.970
2.2	9.025	.111	4.457	4.568	.976
2.3	9.974	.100	4.937	5.037	.980
2.4	11.02	.091	5.466	5.557	.984
2.5	12.18	.082	6.050	6.132	.987
2.6	13.46	.074	6.695	6.769	.989
2.7	14.88	.067	7.406	7.473	.991
2.8	16.44	.061	8.192	8.253	.993
2.9	18.17	.055	9.060	9.115	.994
3.0	20.09	.050	10.018	10.068	.995
3.5	33.12	.030	16.543	16.573	.998
4.0	54.60	.018	27.290	27.308	.999
4.5	90.02	.011	45.003	45.014	1.000
5.0	148.4	.007	74.203	74.210	1.000
6.0	403.4	.002	201.71	201.72	1.000
7.0	1096.6	.001	548.30	548.30	1.000
8.0	2981.0	.000	1490.5	1490.5	1.000
9.0	8103.1	.000	4051.5	4051.5	1.000
10.0	22026.	.000	11013.	11013.	1.000

COLLECTION OF FORMULAS

FORMULAS FROM ALGEBRA AND GEOMETRY

1. Binomial Theorem (n being a positive integer):

$$(a+b)^n = a^n + na^{n-1}b + \frac{n(n-1)}{[2]} a^{n-2}b^2 + \frac{n(n-1)(n-2)}{[3]} a^{n-3}b^3 + \dots \\ + \frac{n(n-1)(n-2) \dots (n-r+2)}{[r-1]} a^{n-r+1}b^{r-1} + \dots$$

2. $n! = [n = 1 \cdot 2 \cdot 3 \cdot 4 \dots (n-1) n]$.

3. The solution of the quadratic equation $ax^2 + bx + c = 0$ is

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

4. When a quadratic equation is reduced to the form $x^2 + px + q = 0$,
 p = sum of roots with sign changed, and q = product of roots.

5. In an arithmetical series,

$$l = a + (n-1)d; s = \frac{n}{2}(a+l) = \frac{n}{2}[2a + (n-1)d].$$

6. In a geometrical series,

$$l = ar^{n-1}; s = \frac{rl - a}{r - 1} = \frac{a(r^n - 1)}{r - 1}.$$

7. Circumference of circle = $2\pi r$.*

8. Area of circle = πr^2 . Area of sector = $\frac{1}{2}r^2\alpha$ (radians).

9. Volume of prism = Bh .

10. Volume of pyramid = $\frac{1}{3}Bh$. Frustum = $\frac{h}{3}(B + B' + \sqrt{BB'})$.

11. Volume of right circular cylinder = πr^2h .

12. Lateral surface of right circular cylinder = $2\pi rh$.

13. Total surface of right circular cylinder = $2\pi r(r+h)$.

14. Volume of right circular cone = $\frac{1}{3}\pi r^2h$.

15. Lateral surface of right circular cone = πrs .

* In formulas 7-20, r denotes radius, h altitude, B area of base, s slant height, and a, b, c the semiaxes of an ellipse or ellipsoid.

16. Total surface of right circular cone $= \pi r(r + s)$.

17. Volume of sphere $= \frac{4}{3} \pi r^3$.

18. Surface of sphere $= 4 \pi r^2$.

19. Area of ellipse $= \pi ab$.

20. Volume of ellipsoid $= \frac{4}{3} \pi abc$.

FORMULAS FROM TRIGONOMETRY

$$21. \cot x = \frac{1}{\tan x}; \sec x = \frac{1}{\cos x}; \csc x = \frac{1}{\sin x}.$$

$$22. \tan x = \frac{\sin x}{\cos x}; \cot x = \frac{\cos x}{\sin x}.$$

$$23. \sin^2 x + \cos^2 x = 1; 1 + \tan^2 x = \sec^2 x; 1 + \cot^2 x = \csc^2 x.$$

$$24. \begin{aligned} \sin(\pi - x) &= \sin x; \sin(\pi + x) = -\sin x; \\ \cos(\pi - x) &= -\cos x; \cos(\pi + x) = -\cos x; \\ \tan(\pi - x) &= -\tan x; \tan(\pi + x) = \tan x. \end{aligned}$$

$$25. \begin{aligned} \sin\left(\frac{\pi}{2} - x\right) &= \cos x; \sin\left(\frac{\pi}{2} + x\right) = \cos x; \\ \cos\left(\frac{\pi}{2} - x\right) &= \sin x; \cos\left(\frac{\pi}{2} + x\right) = -\sin x; \\ \tan\left(\frac{\pi}{2} - x\right) &= \cot x; \tan\left(\frac{\pi}{2} + x\right) = -\cot x. \end{aligned}$$

$$26. \sin(x + y) = \sin x \cos y + \cos x \sin y.$$

$$27. \sin(x - y) = \sin x \cos y - \cos x \sin y.$$

$$28. \cos(x + y) = \cos x \cos y - \sin x \sin y.$$

$$29. \cos(x - y) = \cos x \cos y + \sin x \sin y.$$

$$30. \tan(x + y) = \frac{\tan x + \tan y}{1 - \tan x \tan y}. \quad 31. \tan(x - y) = \frac{\tan x - \tan y}{1 + \tan x \tan y}.$$

$$32. \sin 2x = 2 \sin x \cos x; \cos 2x = \cos^2 x - \sin^2 x; \tan 2x = \frac{2 \tan x}{1 - \tan^2 x}.$$

$$33. \begin{aligned} \sin x &= 2 \sin \frac{x}{2} \cos \frac{x}{2}; & \tan x &= \frac{2 \tan \frac{x}{2}}{1 - \tan^2 \frac{x}{2}}. \\ \cos x &= \cos^2 \frac{x}{2} - \sin^2 \frac{x}{2}; \end{aligned}$$

$$34. \cos^2 x = \frac{1}{2} + \frac{1}{2} \cos 2x; \sin^2 x = \frac{1}{2} - \frac{1}{2} \cos 2x.$$

$$35. 1 + \cos x = 2 \cos^2 \frac{x}{2}; 1 - \cos x = 2 \sin^2 \frac{x}{2}.$$

$$36. \sin \frac{x}{2} = \pm \sqrt{\frac{1 - \cos x}{2}}; \quad \tan \frac{x}{2} = \pm \sqrt{\frac{1 - \cos x}{1 + \cos x}}.$$

$$\cos \frac{x}{2} = \pm \sqrt{\frac{1 + \cos x}{2}};$$

$$37. \sin x + \sin y = 2 \sin \frac{1}{2}(x + y) \cos \frac{1}{2}(x - y).$$

$$38. \sin x - \sin y = 2 \cos \frac{1}{2}(x + y) \sin \frac{1}{2}(x - y).$$

$$39. \cos x + \cos y = 2 \cos \frac{1}{2}(x + y) \cos \frac{1}{2}(x - y).$$

$$40. \cos x - \cos y = -2 \sin \frac{1}{2}(x + y) \sin \frac{1}{2}(x - y).$$

$$41. \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}; \text{ law of sines. } \text{Area} = \frac{1}{2}bc \sin A.$$

$$42. a^2 = b^2 + c^2 - 2bc \cos A; \text{ law of cosines.}$$

FORMULAS FROM ANALYTIC GEOMETRY

$$43. d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}; \text{ distance between points } (x_1, y_1) \text{ and } (x_2, y_2).$$

$$44. d = \frac{Ax_1 + By_1 + C}{\pm \sqrt{A^2 + B^2}}; \text{ distance from line } Ax + By + C = 0 \text{ to } (x_1, y_1).$$

$$45. x = \frac{x_1 + \lambda x_2}{1 + \lambda}, y = \frac{y_1 + \lambda y_2}{1 + \lambda}; (x, y) \text{ is the point dividing the line } P_1P_2 \text{ in the ratio } \lambda.$$

$$46. x = x_0 + x', y = y_0 + y'; \text{ transforming to new origin } (x_0, y_0).$$

$$47. x = x' \cos \theta - y' \sin \theta, y = x' \sin \theta + y' \cos \theta; \text{ transforming to new axes, making the angle } \theta \text{ with old.}$$

$$48. x = \rho \cos \theta, y = \rho \sin \theta; \text{ transforming from rectangular to polar coördinates.}$$

$$49. \rho = \sqrt{x^2 + y^2}, \theta = \arctan \frac{y}{x}; \text{ transforming from polar to rectangular coördinates.}$$

$$50. \text{Equation of a straight line:}$$

$$(a) Ax + By + C = 0, \text{ general form;}$$

$$(b) y - y_1 = m(x - x_1), \text{ slope-point form;}$$

$$(c) x \cos \alpha + y \sin \alpha = p, \text{ normal form.}$$

$$51. \tan \theta = \frac{m_1 - m_2}{1 + m_1 m_2}; \text{ angle between two lines whose slopes are } m_1 \text{ and } m_2.$$

$$m_1 = m_2 \text{ when lines are parallel;}$$

$$m_1 = -\frac{1}{m_2} \text{ when lines are perpendicular.}$$

52. $(x - \alpha)^2 + (y - \beta)^2 = r^2$; equation of circle with center (α, β) and radius r .

53. Area of the triangle $P_1(x_1, y_1), P_2(x_2, y_2), P_3(x_3, y_3)$ is
 $\frac{1}{2}(x_1y_2 - x_2y_1 + x_2y_3 - x_3y_2 + x_3y_1 - x_1y_3)$.

54. Equation of a parabola with vertex at the origin and focus on the x -axis is $y^2 = 2px$.

55. Equation of an ellipse with center at the origin and foci on the x -axis is $b^2x^2 + a^2y^2 = a^2b^2$.

56. Equation of a hyperbola with center at the origin and foci on the x -axis is $b^2x^2 - a^2y^2 = a^2b^2$.

57. Locus of $Ax^2 + Cy^2 + Dx + Ey + F = 0$ is parabola if $A = 0$ or $C = 0$; ellipse if A and C have same sign; hyperbola if A and C have opposite signs.

FORMULAS FROM CALCULUS

58. Radius of curvature.

Center of curvature.

(a) Rectangular coördinates.

$$R = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}}}{\frac{d^2y}{dx^2}}.$$

$$\alpha = x - \frac{\frac{dy}{dx} \left[1 + \left(\frac{dy}{dx}\right)^2\right]}{\frac{d^2y}{dx^2}},$$

(b) Polar coördinates.

$$R = \frac{\left[\rho^2 + \left(\frac{d\rho}{d\theta}\right)^2\right]^{\frac{3}{2}}}{\rho^2 - \rho \frac{d^2\rho}{d\theta^2} + 2\left(\frac{d\rho}{d\theta}\right)^2}.$$

$$\beta = y + \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]}{\frac{d^2y}{dx^2}}.$$

(c) Parametric form.

$$R = \frac{\left[\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2\right]^{\frac{3}{2}}}{\frac{dx}{dt} \frac{d^2y}{dt^2} - \frac{dy}{dt} \frac{d^2x}{dt^2}}.$$

59. Plane area.

(a) Rectangular coördinates.

$$A = \int y dx = \iint dy dx.$$

(b) Polar coördinates.

$$A = \frac{1}{2} \int \rho^2 d\theta = \iint \rho d\rho d\theta.$$

60. Length of arc.

(a) Rectangular coördinates.

$$s = \int [dx^2 + dy^2]^{\frac{1}{2}} = \int \left[1 + \left(\frac{dy}{dx} \right)^2 \right]^{\frac{1}{2}} dx = \int \left[\left(\frac{dx}{dy} \right)^2 + 1 \right]^{\frac{1}{2}} dy.$$

(b) Polar coördinates.

$$s = \int \left[\rho^2 + \left(\frac{d\rho}{d\theta} \right)^2 \right]^{\frac{1}{2}} d\theta = \int \left[\rho^2 \left(\frac{d\theta}{d\rho} \right)^2 + 1 \right]^{\frac{1}{2}} d\rho.$$

61. Volume of solid of revolution about the X -axis.

$$V = \pi \int y^2 dx.$$

62. Area of surface of revolution about the X -axis.

$$A = 2\pi \int y ds = 2\pi \int y \left[1 + \left(\frac{dy}{dx} \right)^2 \right]^{\frac{1}{2}} dx = 2\pi \int y \left[\left(\frac{dx}{dy} \right)^2 + 1 \right]^{\frac{1}{2}} dy.$$

63. Taylor's Series.

$$f(x) = f(a) + \frac{(x-a)}{[1]} f'(a) + \frac{(x-a)^2}{[2]} f''(a) + \frac{(x-a)^3}{[3]} f'''(a) + \dots$$

64. Maclaurin's Series.

$$f(x) = f(0) + \frac{x}{[1]} f'(0) + \frac{x^2}{[2]} f''(0) + \frac{x^3}{[3]} f'''(0) + \dots$$

65. Standard Series.

Convergent for

$$e^z = 1 + z + \frac{z^2}{[2]} + \frac{z^3}{[3]} + \frac{z^4}{[4]} + \dots,$$

all values of z ,

$$\sin z = z - \frac{z^3}{[3]} + \frac{z^5}{[5]} - \frac{z^7}{[7]} + \dots,$$

all values of z ,

$$\cos z = 1 - \frac{z^2}{[2]} + \frac{z^4}{[4]} - \frac{z^6}{[6]} + \dots,$$

all values of z ,

$$\log(1+z) = z - \frac{z^2}{2} + \frac{z^3}{3} - \frac{z^4}{4} + \dots,$$

 $-1 < z \leq 1$,

$$(1+z)^n = 1 + \frac{n}{1}z + \frac{n(n-1)}{1 \cdot 2}z^2 + \dots,$$

 $-1 < z \leq 1$,

$$\sinh z = z + \frac{z^3}{[3]} + \frac{z^5}{[5]} + \frac{z^7}{[7]} + \dots,$$

all values of z ,

$$\cosh z = 1 + \frac{z^2}{[2]} + \frac{z^4}{[4]} + \frac{z^6}{[6]} + \dots,$$

all values of z .

FORMULAS FOR DIFFERENTIATION

In these formulas u , v , and w denote *variable* quantities which are functions of x .

$$\text{I} \quad \frac{dc}{dx} = 0.$$

$$\text{II} \quad \frac{dx}{dx} = 1.$$

$$\text{III} \quad \frac{d}{dx}(u + v - w) = \frac{du}{dx} + \frac{dv}{dx} - \frac{dw}{dx}.$$

$$\text{IV} \quad \frac{d}{dx}(cv) = c \frac{dv}{dx}.$$

$$\text{V} \quad \frac{d}{dx}(uv) = u \frac{dv}{dx} + v \frac{du}{dx}.$$

$$\text{VI} \quad \frac{d}{dx}(v_1 v_2 \cdots v_n) = (v_2 v_3 \cdots v_n) \frac{dv_1}{dx} + (v_1 v_3 \cdots v_n) \frac{dv_2}{dx} + \cdots + (v_1 v_2 \cdots v_{n-1}) \frac{dv_n}{dx}.$$

$$\text{VII} \quad \frac{d}{dx}(v^n) = n v^{n-1} \frac{dv}{dx}.$$

$$\text{VII } a \quad \frac{d}{dx}(x^n) = n x^{n-1}.$$

$$\text{VIII} \quad \frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}.$$

$$\text{VIII } a \quad \frac{d}{dx}\left(\frac{u}{c}\right) = \frac{\frac{du}{dx}}{c}.$$

$$\text{VIII } b \quad \frac{d}{dx}\left(\frac{c}{v}\right) = -\frac{c \frac{dv}{dx}}{v^2}.$$

$$\text{IX} \quad \frac{d}{dx}(\log_a v) = \log_a e \cdot \frac{\frac{dv}{dx}}{v}.$$

$$\text{IX } a \quad \frac{d}{dx}(\log v) = \frac{\frac{dv}{dx}}{v}.$$

$$\text{X} \quad \frac{d}{dx}(a^v) = a^v \log a \frac{dv}{dx}.$$

$$\text{X } a \quad \frac{d}{dx}(e^v) = e^v \frac{dv}{dx}.$$

$$\text{XI} \quad \frac{d}{dx}(u^v) = v u^{v-1} \frac{du}{dx} + \log u \cdot u^v \frac{dv}{dx}.$$

XII	$\frac{d}{dx} (\sin v) = \cos v \frac{dv}{dx}.$
XIII	$\frac{d}{dx} (\cos v) = -\sin v \frac{dv}{dx}.$
XIV	$\frac{d}{dx} (\tan v) = \sec^2 v \frac{dv}{dx}.$
XV	$\frac{d}{dx} (\cot v) = -\csc^2 v \frac{dv}{dx}.$
XVI	$\frac{d}{dx} (\sec v) = \sec v \tan v \frac{dv}{dx}.$
XVII	$\frac{d}{dx} (\csc v) = -\csc v \cot v \frac{dv}{dx}.$
XVIII	$\frac{d}{dx} (\text{vers } v) = \sin v \frac{dv}{dx}.$
XIX	$\frac{d}{dx} (\text{arc sin } v) = \frac{\frac{dv}{dx}}{\sqrt{1-v^2}}.$
XX	$\frac{d}{dx} (\text{arc cos } v) = -\frac{\frac{dv}{dx}}{\sqrt{1-v^2}}.$
XXI	$\frac{d}{dx} (\text{arc tan } v) = \frac{\frac{dv}{dx}}{1+v^2}.$
XXII	$\frac{d}{dx} (\text{arc cot } v) = -\frac{\frac{dv}{dx}}{1+v^2}.$
XXIII	$\frac{d}{dx} (\text{arc sec } v) = \frac{\frac{dv}{dx}}{v \sqrt{v^2-1}}.$
XXIV	$\frac{d}{dx} (\text{arc csc } v) = -\frac{\frac{dv}{dx}}{v \sqrt{v^2-1}}.$
XXV	$\frac{d}{dx} (\text{arc vers } v) = \frac{\frac{dv}{dx}}{\sqrt{2v-v^2}}.$
XXVI	$\frac{dy}{dx} = \frac{dy}{dv} \cdot \frac{dv}{dx}, \text{ } y \text{ being a function of } v.$
XXVII	$\frac{dy}{dx} = \frac{1}{\frac{dx}{dy}}, \text{ } y \text{ being a function of } x.$

TABLE OF INTEGRALS

SOME ELEMENTARY FORMS

$$1. \int (du \pm dv \pm dw \pm \dots) = \int du \pm \int dv \pm \int dw \pm \dots$$

$$2. \int a dv = a \int dv.$$

$$4. \int x^n dx = \frac{x^{n+1}}{n+1} + C, n \neq -1.$$

$$3. \int df(x) = \int f'(x) dx = f(x) + C.$$

$$5. \int \frac{dx}{x} = \log x + C.$$

FORMS CONTAINING INTEGRAL POWERS OF $a + bx$

$$6. \int \frac{dx}{a + bx} = \frac{1}{b} \log(a + bx) + C.$$

$$7. \int (a + bx)^n dx = \frac{(a + bx)^{n+1}}{b(n+1)} + C, n \neq -1.$$

$$8. \int F(x, a + bx) dx. \text{ Try one of the substitutions, } z = a + bx, xz = a + bx.$$

$$9. \int \frac{x dx}{a + bx} = \frac{1}{b^2} [a + bx - a \log(a + bx)] + C.$$

$$10. \int \frac{x^2 dx}{a + bx} = \frac{1}{b^3} \left[\frac{1}{2} (a + bx)^2 - 2a(a + bx) + a^2 \log(a + bx) \right] + C.$$

$$11. \int \frac{dx}{x(a + bx)} = -\frac{1}{a} \log \frac{a + bx}{x} + C.$$

$$12. \int \frac{dx}{x^2(a + bx)} = -\frac{1}{ax} + \frac{b}{a^2} \log \frac{a + bx}{x} + C.$$

$$13. \int \frac{x dx}{(a + bx)^2} = \frac{1}{b^2} \left[\log(a + bx) + \frac{a}{a + bx} \right] + C.$$

$$14. \int \frac{x^2 dx}{(a + bx)^2} = \frac{1}{b^3} \left[a + bx - 2a \log(a + bx) - \frac{a^2}{a + bx} \right] + C.$$

$$15. \int \frac{dx}{x(a + bx)^2} = \frac{1}{a(a + bx)} - \frac{1}{a^2} \log \frac{a + bx}{x} + C.$$

$$16. \int \frac{x dx}{(a + bx)^3} = \frac{1}{b^2} \left[-\frac{1}{a + bx} + \frac{a}{2(a + bx)^2} \right] + C.$$

FORMS CONTAINING $a^2 + x^2$, $a^2 - x^2$, $a + bx^2$, $a + bx^n$

17. $\int \frac{dx}{a^2 + x^2} = \frac{1}{a} \tan^{-1} \frac{x}{a} + C$; $\int \frac{dx}{1 + x^2} = \tan^{-1} x + C$.
18. $\int \frac{dx}{a^2 - x^2} = \frac{1}{2a} \log \frac{a+x}{a-x} + C$; $\int \frac{dx}{x^2 - a^2} = \frac{1}{2a} \log \frac{x-a}{x+a} + C$.
19. $\int \frac{dx}{a + bx^2} = \frac{1}{\sqrt{ab}} \tan^{-1} x \sqrt{\frac{b}{a}} + C$, when $a > 0$ and $b > 0$.
20. $\int \frac{dx}{a^2 - b^2 x^2} = \frac{1}{2ab} \log \frac{a+bx}{a-bx} + C$.
21. $\int x^m (a + bx^n)^p dx$
 $= \frac{x^{m-n+1} (a + bx^n)^{p+1}}{b(np + m + 1)} - \frac{a(m - n + 1)}{b(np + m + 1)} \int x^{m-n} (a + bx^n)^p dx$.
22. $\int x^m (a + bx^n)^p dx = \frac{x^{m+1} (a + bx^n)^p}{np + m + 1} + \frac{anp}{np + m + 1} \int x^m (a + bx^n)^{p-1} dx$.
23. $\int \frac{dx}{x^m (a + bx^n)^p} = -\frac{1}{(m-1)ax^{m-1}(a + bx^n)^{p-1}}$
 $- \frac{(m-n+np-1)b}{(m-1)a} \int \frac{dx}{x^{m-n}(a + bx^n)^p}$.
24. $\int \frac{dx}{x^m (a + bx^n)^p}$
 $= \frac{1}{an(p-1)x^{m-1}(a + bx^n)^{p-1}} + \frac{m-n+np-1}{an(p-1)} \int \frac{dx}{x^m (a + bx^n)^{p-1}}$.
25. $\int \frac{(a + bx^n)^p dx}{x^m} = -\frac{(a + bx^n)^{p+1}}{a(m-1)x^{m-1}}$
 $- \frac{b(m-n-np-1)}{a(m-1)} \int \frac{(a + bx^n)^p dx}{x^{m-n}}$.
26. $\int \frac{(a + bx^n)^p dx}{x^m} = \frac{(a + bx^n)^p}{(np-m+1)x^{m-1}} + \frac{anp}{np-m+1} \int \frac{(a + bx^n)^{p-1} dx}{x^m}$.
27. $\int \frac{x^m dx}{(a + bx^n)^p} = \frac{x^{m-n+1}}{b(m-np+1)(a + bx^n)^{p-1}}$
 $- \frac{a(m-n+1)}{b(m-np+1)} \int \frac{x^{m-n} dx}{(a + bx^n)^p}$.

- $$28. \int \frac{x^m dx}{(a + bx^n)^p} = \frac{x^{m+1}}{an(p-1)(a + bx^n)^{p-1}} - \frac{m+n-np+1}{an(p-1)} \int \frac{x^m dx}{(a + bx^n)^{p-1}}.$$
- $$29. \int \frac{dx}{(a^2 + x^2)^n} = \frac{1}{2(n-1)a^2} \left[\frac{x}{(a^2 + x^2)^{n-1}} + (2n-3) \int \frac{dx}{(a^2 + x^2)^{n-1}} \right].$$
- $$30. \int \frac{dx}{(a + bx^2)^n} = \frac{1}{2(n-1)a} \left[\frac{x}{(a + bx^2)^{n-1}} + (2n-3) \int \frac{dx}{(a + bx^2)^{n-1}} \right].$$
- $$31. \int \frac{x dx}{(a + bx^2)^n} = \frac{1}{2} \int \frac{dz}{(a + bz)^n}, \text{ where } z = x^2.$$
- $$32. \int \frac{x^2 dx}{(a + bx^2)^n} = \frac{-x}{2b(n-1)(a + bx^2)^{n-1}} + \frac{1}{2b(n-1)} \int \frac{dx}{(a + bx^2)^{n-1}}.$$
- $$33. \int \frac{dx}{x(a + bx^n)} = \frac{1}{an} \log \frac{x^n}{a + bx^n} + C.$$
- $$34. \int \frac{dx}{x^2(a + bx^2)^n} = \frac{1}{a} \int \frac{dx}{x^2(a + bx^2)^{n-1}} - \frac{b}{a} \int \frac{dx}{(a + bx^2)^n}.$$
- $$35. \int \frac{x dx}{a + bx^2} = \frac{1}{2b} \log \left(x^2 + \frac{a}{b} \right) + C.$$
- $$36. \int \frac{x^2 dx}{a + bx^2} = \frac{x}{b} - \frac{a}{b} \int \frac{dx}{a + bx^2}.$$
- $$37. \int \frac{dx}{x(a + bx^2)} = \frac{1}{2a} \log \frac{x^2}{a + bx^2} + C.$$
- $$38. \int \frac{dx}{x^2(a + bx^2)} = -\frac{1}{ax} - \frac{b}{a} \int \frac{dx}{a + bx^2}.$$
- $$39. \int \frac{dx}{(a + bx^2)^2} = \frac{x}{2a(a + bx^2)} + \frac{1}{2a} \int \frac{dx}{a + bx^2}.$$

FORMS CONTAINING $\sqrt{a + bx}$.

- $$40. \int x \sqrt{a + bx} dx = -\frac{2(2a - 3bx) \sqrt{(a + bx)^3}}{15b^2} + C.$$
- $$41. \int x^2 \sqrt{a + bx} dx = \frac{2(8a^2 - 12abx + 15b^2x^2) \sqrt{(a + bx)^3}}{105b^3} + C.$$
- $$42. \int \frac{x dx}{\sqrt{a + bx}} = -\frac{2(2a - bx)}{3b^2} \sqrt{a + bx} + C.$$

$$43. \int \frac{x^2 dx}{\sqrt{a+bx}} = \frac{2(8a^2 - 4abx + 3b^2x^2)}{15b^3} \sqrt{a+bx} + C.$$

$$44. \int \frac{dx}{x\sqrt{a+bx}} = \frac{1}{\sqrt{a}} \log \frac{\sqrt{a+bx} - \sqrt{a}}{\sqrt{a+bx} + \sqrt{a}} + C, \text{ for } a > 0.$$

$$45. \int \frac{dx}{x\sqrt{a+bx}} = \frac{2}{\sqrt{-a}} \tan^{-1} \sqrt{\frac{a+bx}{-a}} + C, \text{ for } a < 0.$$

$$46. \int \frac{dx}{x^2\sqrt{a+bx}} = \frac{-\sqrt{a+bx}}{ax} - \frac{b}{2a} \int \frac{dx}{x\sqrt{a+bx}}.$$

$$47. \int \frac{\sqrt{a+bx} dx}{x} = 2\sqrt{a+bx} + a \int \frac{dx}{x\sqrt{a+bx}}.$$

FORMS CONTAINING $\sqrt{x^2+a^2}$

$$48. \int (x^2+a^2)^{\frac{1}{2}} dx = \frac{x}{2} \sqrt{x^2+a^2} + \frac{a^2}{2} \log(x+\sqrt{x^2+a^2}) + C.$$

$$49. \int (x^2+a^2)^{\frac{3}{2}} dx = \frac{x}{8} (2x^2+5a^2) \sqrt{x^2+a^2} + \frac{3a^4}{8} \log(x+\sqrt{x^2+a^2}) + C.$$

$$50. \int (x^2+a^2)^{\frac{n}{2}} dx = \frac{x(x^2+a^2)^{\frac{n}{2}}}{n+1} + \frac{na^2}{n+1} \int (x^2+a^2)^{\frac{n}{2}-1} dx.$$

$$51. \int x(x^2+a^2)^{\frac{n}{2}} dx = \frac{(x^2+a^2)^{\frac{n+2}{2}}}{n+2} + C.$$

$$52. \int x^2(x^2+a^2)^{\frac{1}{2}} dx = \frac{x}{8} (2x^2+a^2) \sqrt{x^2+a^2} - \frac{a^4}{8} \log(x+\sqrt{x^2+a^2}) + C.$$

$$53. \int \frac{dx}{(x^2+a^2)^{\frac{1}{2}}} = \log(x+\sqrt{x^2+a^2}) + C.$$

$$54. \int \frac{dx}{(x^2+a^2)^{\frac{3}{2}}} = \frac{x}{a^2\sqrt{x^2+a^2}} + C.$$

$$55. \int \frac{xdx}{(x^2+a^2)^{\frac{1}{2}}} = \sqrt{x^2+a^2} + C.$$

$$56. \int \frac{x^2 dx}{(x^2+a^2)^{\frac{1}{2}}} = \frac{x}{2} \sqrt{x^2+a^2} - \frac{a^2}{2} \log(x+\sqrt{x^2+a^2}) + C.$$

$$57. \int \frac{x^2 dx}{(x^2 + a^2)^{\frac{3}{2}}} = -\frac{x}{\sqrt{x^2 + a^2}} + \log(x + \sqrt{x^2 + a^2}) + C.$$

$$58. \int \frac{dx}{x(x^2 + a^2)^{\frac{1}{2}}} = \frac{1}{a} \log \frac{x}{a + \sqrt{x^2 + a^2}} + C.$$

$$59. \int \frac{dx}{x^2(x^2 + a^2)^{\frac{1}{2}}} = -\frac{\sqrt{x^2 + a^2}}{a^2 x} + C.$$

$$60. \int \frac{dx}{x^3(x^2 + a^2)^{\frac{1}{2}}} = -\frac{\sqrt{x^2 + a^2}}{2a^2 x^2} + \frac{1}{2a^3} \log \frac{a + \sqrt{x^2 + a^2}}{x} + C.$$

$$61. \int \frac{(x^2 + a^2)^{\frac{1}{2}} dx}{x} = \sqrt{a^2 + x^2} - a \log \frac{a + \sqrt{a^2 + x^2}}{x} + C.$$

$$62. \int \frac{(x^2 + a^2)^{\frac{1}{2}} dx}{x^2} = -\frac{\sqrt{x^2 + a^2}}{x} + \log(x + \sqrt{x^2 + a^2}) + C.$$

FORMS CONTAINING $\sqrt{x^2 - a^2}$

$$63. \int (x^2 - a^2)^{\frac{1}{2}} dx = \frac{x}{2} \sqrt{x^2 - a^2} - \frac{a^2}{2} \log(x + \sqrt{x^2 - a^2}) + C.$$

$$64. \int (x^2 - a^2)^{\frac{3}{2}} dx = \frac{x}{8} (2x^2 - 5a^2) \sqrt{x^2 - a^2} + \frac{3a^4}{8} \log(x + \sqrt{x^2 - a^2}) + C.$$

$$65. \int (x^2 - a^2)^{\frac{n}{2}} dx = \frac{x(x^2 - a^2)^{\frac{n}{2}}}{n+1} - \frac{na^2}{n+1} \int (x^2 - a^2)^{\frac{n}{2}-1} dx.$$

$$66. \int x(x^2 - a^2)^{\frac{n}{2}} dx = \frac{(x^2 - a^2)^{\frac{n+2}{2}}}{n+2} + C.$$

$$67. \int x^2(x^2 - a^2)^{\frac{1}{2}} dx = \frac{x}{8} (2x^2 - a^2) \sqrt{x^2 - a^2} - \frac{a^4}{8} \log(x + \sqrt{x^2 - a^2}) + C.$$

$$68. \int \frac{dx}{(x^2 - a^2)^{\frac{1}{2}}} = \log(x + \sqrt{x^2 - a^2}) + C.$$

$$69. \int \frac{dx}{(x^2 - a^2)^{\frac{3}{2}}} = -\frac{x}{a^2 \sqrt{x^2 - a^2}} + C.$$

$$70. \int \frac{xdx}{(x^2 - a^2)^{\frac{1}{2}}} = \sqrt{x^2 - a^2} + C.$$

71. $\int \frac{x^2 dx}{(x^2 - a^2)^{\frac{1}{2}}} = \frac{x}{2} \sqrt{x^2 - a^2} + \frac{a^2}{2} \log(x + \sqrt{x^2 - a^2}) + C.$
72. $\int \frac{x^2 dx}{(x^2 - a^2)^{\frac{3}{2}}} = -\frac{x}{\sqrt{x^2 - a^2}} + \log(x + \sqrt{x^2 - a^2}) + C.$
73. $\int \frac{dx}{x(x^2 - a^2)^{\frac{1}{2}}} = \frac{1}{a} \sec^{-1} \frac{x}{a} + C; \int \frac{dx}{x \sqrt{x^2 - 1}} = \sec^{-1} x + C.$
74. $\int \frac{dx}{x^2(x^2 - a^2)^{\frac{1}{2}}} = \frac{\sqrt{x^2 - a^2}}{a^2 x} + C.$
75. $\int \frac{dx}{x^3(x^2 - a^2)^{\frac{1}{2}}} = \frac{\sqrt{x^2 - a^2}}{2 a^2 x^2} + \frac{1}{2 a^3} \sec^{-1} \frac{x}{a} + C.$
76. $\int \frac{(x^2 - a^2)^{\frac{1}{2}} dx}{x} = \sqrt{x^2 - a^2} - a \cos^{-1} \frac{a}{x} + C.$
77. $\int \frac{(x^2 - a^2)^{\frac{1}{2}} dx}{x^2} = -\frac{\sqrt{x^2 - a^2}}{x} + \log(x + \sqrt{x^2 - a^2}) + C.$

FORMS CONTAINING $\sqrt{a^2 - x^2}$

78. $\int (a^2 - x^2)^{\frac{1}{2}} dx = \frac{x}{2} \sqrt{a^2 - x^2} + \frac{a^2}{2} \sin^{-1} \frac{x}{a} + C.$
79. $\int (a^2 - x^2)^{\frac{3}{2}} dx = \frac{x}{8} (5a^2 - 2x^2) \sqrt{a^2 - x^2} + \frac{3a^4}{8} \sin^{-1} \frac{x}{a} + C.$
80. $\int (a^2 - x^2)^{\frac{n}{2}} dx = \frac{x(a^2 - x^2)^{\frac{n}{2}}}{n+1} + \frac{a^2 n}{n+1} \int (a^2 - x^2)^{\frac{n}{2}-1} dx.$
81. $\int x(a^2 - x^2)^{\frac{n}{2}} dx = -\frac{(a^2 - x^2)^{\frac{n+2}{2}}}{n+2} + C.$
82. $\int x^2(a^2 - x^2)^{\frac{1}{2}} dx = \frac{x}{8} (2x^2 - a^2) \sqrt{a^2 - x^2} + \frac{a^4}{8} \sin^{-1} \frac{x}{a} + C.$
83. $\int \frac{dx}{(a^2 - x^2)^{\frac{1}{2}}} = \sin^{-1} \frac{x}{a} + C; \int \frac{dx}{\sqrt{1 - x^2}} = \sin^{-1} x + C.$
84. $\int \frac{dx}{(a^2 - x^2)^{\frac{3}{2}}} = \frac{x}{a^2 \sqrt{a^2 - x^2}} + C.$
85. $\int \frac{xdx}{(a^2 - x^2)^{\frac{1}{2}}} = -\sqrt{a^2 - x^2} + C.$

86. $\int \frac{x^2 dx}{(a^2 - x^2)^{\frac{1}{2}}} = -\frac{x}{2} \sqrt{a^2 - x^2} + \frac{a^2}{2} \sin^{-1} \frac{x}{a} + C.$
87. $\int \frac{x^2 dx}{(a^2 - x^2)^{\frac{3}{2}}} = \frac{x}{\sqrt{a^2 - x^2}} - \sin^{-1} \frac{x}{a} + C.$
88. $\int \frac{x^m dx}{(a^2 - x^2)^{\frac{1}{2}}} = -\frac{x^{m-1}}{m} \sqrt{a^2 - x^2} + \frac{(m-1)a^2}{m} \int \frac{x^{m-2}}{(a^2 - x^2)^{\frac{1}{2}}} dx.$
89. $\int \frac{dx}{x(a^2 - x^2)^{\frac{1}{2}}} = \frac{1}{a} \log \frac{x}{a + \sqrt{a^2 - x^2}} + C.$
90. $\int \frac{dx}{x^2(a^2 - x^2)^{\frac{1}{2}}} = -\frac{\sqrt{a^2 - x^2}}{a^2 x} + C.$
91. $\int \frac{dx}{x^3(a^2 - x^2)^{\frac{1}{2}}} = -\frac{\sqrt{a^2 - x^2}}{2a^2 x^2} + \frac{1}{2a^3} \log \frac{x}{a + \sqrt{a^2 - x^2}} + C.$
92. $\int \frac{(a^2 - x^2)^{\frac{1}{2}}}{x} dx = \sqrt{a^2 - x^2} - a \log \frac{a + \sqrt{a^2 - x^2}}{x} + C.$
93. $\int \frac{(a^2 - x^2)^{\frac{1}{2}}}{x^2} dx = -\frac{\sqrt{a^2 - x^2}}{x} - \sin^{-1} \frac{x}{a} + C.$

FORMS CONTAINING $\sqrt{2ax - x^2}$, $\sqrt{2ax + x^2}$

94. $\int \sqrt{2ax - x^2} dx = \frac{x-a}{2} \sqrt{2ax - x^2} + \frac{a^2}{2} \text{vers}^{-1} \frac{x}{a} + C.$
95. $\int \frac{dx}{\sqrt{2ax - x^2}} = \text{vers}^{-1} \frac{x}{a} + C; \int \frac{dx}{\sqrt{2ax + x^2}} = \text{vers}^{-1} x + C.$
96. $\int x^m \sqrt{2ax - x^2} dx = -\frac{x^{m-1}(2ax - x^2)^{\frac{3}{2}}}{m+2} + \frac{(2m+1)a}{m+2} \int x^{m-1} \sqrt{2ax - x^2} dx.$
97. $\int \frac{dx}{x^m \sqrt{2ax - x^2}} = -\frac{\sqrt{2ax - x^2}}{(2m-1)ax^m} + \frac{m-1}{(2m-1)a} \int \frac{dx}{x^{m-1} \sqrt{2ax - x^2}}.$
98. $\int \frac{x^m dx}{\sqrt{2ax - x^2}} = -\frac{x^{m-1} \sqrt{2ax - x^2}}{m} + \frac{(2m-1)a}{m} \int \frac{x^{m-1} dx}{\sqrt{2ax - x^2}}.$

99. $\int \frac{\sqrt{2ax-x^2}}{x^m} dx = -\frac{(2ax-x^2)^{\frac{3}{2}}}{(2m-3)ax^m} + \frac{m-3}{(2m-3)a} \int \frac{\sqrt{2ax-x^2}}{x^{m-1}} dx.$
100. $\int x \sqrt{2ax-x^2} dx = -\frac{3a^2+ax-2x^2}{6} \sqrt{2ax-x^2} + \frac{a^3}{2} \text{vers}^{-1} \frac{x}{a}.$
101. $\int \frac{dx}{x \sqrt{2ax-x^2}} = -\frac{\sqrt{2ax-x^2}}{ax} + C.$
102. $\int \frac{x dx}{\sqrt{2ax-x^2}} = -\sqrt{2ax-x^2} + a \text{vers}^{-1} \frac{x}{a} + C.$
103. $\int \frac{x^2 dx}{\sqrt{2ax-x^2}} = -\frac{x+3a}{2} \sqrt{2ax-x^2} + \frac{3}{2} a^2 \text{vers}^{-1} \frac{x}{a} + C.$
104. $\int \frac{\sqrt{2ax-x^2}}{x} dx = \sqrt{2ax-x^2} + a \text{vers}^{-1} \frac{x}{a} + C.$
105. $\int \frac{\sqrt{2ax-x^2}}{x^2} dx = -\frac{2\sqrt{2ax-x^2}}{x} - \text{vers}^{-1} \frac{x}{a} + C.$
106. $\int \frac{\sqrt{2ax-x^2}}{x^3} dx = -\frac{(2ax-x^2)^{\frac{3}{2}}}{3ax^3} + C.$
107. $\int \frac{dx}{(2ax-x^2)^{\frac{3}{2}}} = \frac{x-a}{a^2 \sqrt{2ax-x^2}} + C.$
108. $\int \frac{x dx}{(2ax-x^2)^{\frac{3}{2}}} = \frac{x}{a \sqrt{2ax-x^2}} + C.$
109. $\int F(x, \sqrt{2ax-x^2}) dx = \int F(z+a, \sqrt{a^2-z^2}) dz, \text{ where } z = x-a.$
110. $\int \frac{dx}{\sqrt{2ax+x^2}} = \log(x+a+\sqrt{2ax+x^2}) + C.$
111. $\int F(x, \sqrt{2ax+x^2}) dx = \int F(z-a, \sqrt{z^2-a^2}) dz, \text{ where } z = x+a.$

FORMS CONTAINING $a+bx \pm cx^2$

112. $\int \frac{dx}{a+bx+cx^2} = \frac{2}{\sqrt{4ac-b^2}} \tan^{-1} \frac{2cx+b}{\sqrt{4ac-b^2}} + C, \text{ when } b^2 < 4ac.$
113. $\int \frac{dx}{a+bx+cx^2} = \frac{1}{\sqrt{b^2-4ac}} \log \frac{2cx+b-\sqrt{b^2-4ac}}{2cx+b+\sqrt{b^2-4ac}} + C,$
when $b^2 > 4ac.$

114. $\int \frac{dx}{a+bx-cx^2} = \frac{1}{\sqrt{b^2+4ac}} \log \frac{\sqrt{b^2+4ac}+2cx-b}{\sqrt{b^2+4ac}-2cx+b} + C.$
115. $\int \frac{dx}{\sqrt{a+bx+cx^2}} = \frac{1}{\sqrt{c}} \log (2cx+b+2\sqrt{c}\sqrt{a+bx+cx^2}) + C.$
116. $\int \sqrt{a+bx+cx^2} dx = \frac{2cx+b}{4c} \sqrt{a+bx+cx^2}$
 $- \frac{b^2-4ac}{8c^{\frac{3}{2}}} \log (2cx+b+2\sqrt{c}\sqrt{a+bx+cx^2}) + C.$
117. $\int \frac{dx}{\sqrt{a+bx-cx^2}} = \frac{1}{\sqrt{c}} \sin^{-1} \frac{2cx-b}{\sqrt{b^2+4ac}} + C.$
118. $\int \sqrt{a+bx-cx^2} dx = \frac{2cx-b}{4c} \sqrt{a+bx-cx^2}$
 $+ \frac{b^2+4ac}{8c^{\frac{3}{2}}} \sin^{-1} \frac{2cx-b}{\sqrt{b^2+4ac}} + C.$
119. $\int \frac{xdx}{\sqrt{a+bx+cx^2}} = \frac{\sqrt{a+bx+cx^2}}{c}$
 $- \frac{b}{2c^{\frac{3}{2}}} \log (2cx+b+2\sqrt{c}\sqrt{a+bx+cx^2}) + C.$
120. $\int \frac{xdx}{\sqrt{a+bx-cx^2}} = -\frac{\sqrt{a+bx-cx^2}}{c} + \frac{b}{2c^{\frac{3}{2}}} \sin^{-1} \frac{2cx-b}{\sqrt{b^2+4ac}} + C.$

OTHER ALGEBRAIC FORMS

121. $\int \sqrt{\frac{a+x}{b+x}} dx = \sqrt{(a+x)(b+x)}$
 $+ (a-b) \log (\sqrt{a+x} + \sqrt{b+x}) + C.$
122. $\int \sqrt{\frac{a-x}{b+x}} dx = \sqrt{(a-x)(b+x)} + (a+b) \sin^{-1} \sqrt{\frac{x+b}{a+b}} + C.$
123. $\int \sqrt{\frac{a+x}{b-x}} dx = -\sqrt{(a+x)(b-x)} - (a+b) \sin^{-1} \sqrt{\frac{b-x}{a+b}} + C.$
124. $\int \sqrt{\frac{1+x}{1-x}} dx = -\sqrt{1-x^2} + \sin^{-1} x + C.$
125. $\int \frac{dx}{\sqrt{(x-\alpha)(\beta-x)}} = 2 \sin^{-1} \sqrt{\frac{x-\alpha}{\beta-\alpha}} + C.$

EXPONENTIAL AND TRIGONOMETRIC FORMS

$$126. \int a^x dx = \frac{a^x}{\log a} + C. \quad 129. \int \sin x dx = -\cos x + C.$$

$$127. \int e^x dx = e^x + C. \quad 130. \int \cos x dx = \sin x + C.$$

$$128. \int e^{ax} dx = \frac{e^{ax}}{a} + C. \quad 131. \int \tan x dx = \log \sec x = -\log \cos x + C.$$

$$132. \int \cot x dx = \log \sin x + C.$$

$$133. \int \sec x dx = \int \frac{dx}{\cos x} = \log (\sec x + \tan x) = \log \tan \left(\frac{\pi}{4} + \frac{x}{2} \right) + C.$$

$$134. \int \operatorname{cosec} x dx = \int \frac{dx}{\sin x} = \log (\operatorname{cosec} x - \cot x) = \log \tan \frac{x}{2} + C.$$

$$135. \int \sec^2 x dx = \tan x + C. \quad 138. \int \operatorname{cosec} x \cot x dx = -\operatorname{cosec} x + C.$$

$$136. \int \operatorname{cosec}^2 x dx = -\cot x + C. \quad 139. \int \sin^2 x dx = \frac{x}{2} - \frac{1}{4} \sin 2x + C.$$

$$137. \int \sec x \tan x dx = \sec x + C. \quad 140. \int \cos^2 x dx = \frac{x}{2} + \frac{1}{4} \sin 2x + C.$$

$$141. \int \sin^n x dx = -\frac{\sin^{n-1} x \cos x}{n} + \frac{n-1}{n} \int \sin^{n-2} x dx.$$

$$142. \int \cos^n x dx = \frac{\cos^{n-1} x \sin x}{n} + \frac{n-1}{n} \int \cos^{n-2} x dx.$$

$$143. \int \frac{dx}{\sin^n x} = -\frac{1}{n-1} \frac{\cos x}{\sin^{n-1} x} + \frac{n-2}{n-1} \int \frac{dx}{\sin^{n-2} x}.$$

$$144. \int \frac{dx}{\cos^n x} = \frac{1}{n-1} \frac{\sin x}{\cos^{n-1} x} + \frac{n-2}{n-1} \int \frac{dx}{\cos^{n-2} x}.$$

$$145. \int \cos^m x \sin^n x dx = \frac{\cos^{m-1} x \sin^{n+1} x}{m+n} + \frac{m-1}{m+n} \int \cos^{m-2} x \sin^n x dx.$$

$$146. \int \cos^m x \sin^n x dx = -\frac{\sin^{n-1} x \cos^{m+1} x}{m+n} + \frac{n-1}{m+n} \int \cos^m x \sin^{n-2} x dx.$$

$$147. \int \frac{dx}{\sin^m x \cos^n x} = \frac{1}{n-1} \cdot \frac{1}{\sin^{m-1} x \cos^{n-1} x} + \frac{m+n-2}{n-1} \int \frac{dx}{\sin^m x \cos^{n-2} x}.$$

- $$148. \int \frac{dx}{\sin^m x \cos^n x} = -\frac{1}{m-1} \cdot \frac{1}{\sin^{m-1} x \cos^{n-1} x} + \frac{m+n-2}{m-1} \int \frac{dx}{\sin^{m-2} x \cos^n x}.$$
- $$149. \int \frac{\cos^m x dx}{\sin^n x} = -\frac{\cos^{m+1} x}{(n-1) \sin^{n-1} x} - \frac{m-n+2}{n-1} \int \frac{\cos^m x dx}{\sin^{n-2} x}.$$
- $$150. \int \frac{\cos^m x dx}{\sin^n x} = \frac{\cos^{m-1} x}{(m-n) \sin^{n-1} x} + \frac{m-1}{m-n} \int \frac{\cos^{m-2} x dx}{\sin^n x}.$$
- $$151. \int \sin x \cos^n x dx = -\frac{\cos^{n+1} x}{n+1} + C.$$
- $$152. \int \sin^n x \cos x dx = \frac{\sin^{n+1} x}{n+1} + C.$$
- $$153. \int \tan^n x dx = \frac{\tan^{n-1} x}{n-1} - \int \tan^{n-2} x dx.$$
- $$154. \int \cot^n x dx = -\frac{\cot^{n-1} x}{n-1} - \int \cot^{n-2} x dx.$$
- $$155. \int \sin mx \sin nx dx = -\frac{\sin(m+n)x}{2(m+n)} + \frac{\sin(m-n)x}{2(m-n)} + C.$$
- $$156. \int \cos mx \cos nx dx = \frac{\sin(m+n)x}{2(m+n)} + \frac{\sin(m-n)x}{2(m-n)} + C.$$
- $$157. \int \sin mx \cos nx dx = -\frac{\cos(m+n)x}{2(m+n)} - \frac{\cos(m-n)x}{2(m-n)} + C.$$
- $$158. \int \frac{dx}{a+b \cos x} = \frac{2}{\sqrt{a^2-b^2}} \tan^{-1} \left(\sqrt{\frac{a-b}{a+b}} \tan \frac{x}{2} \right) + C, \text{ when } a > b.$$
- $$159. \int \frac{dx}{a+b \cos x} = \frac{1}{\sqrt{b^2-a^2}} \log \frac{\sqrt{b-a} \tan \frac{x}{2} + \sqrt{b+a}}{\sqrt{b-a} \tan \frac{x}{2} - \sqrt{b+a}} + C, \text{ when } a < b.$$
- $$160. \int \frac{dx}{a+b \sin x} = \frac{2}{\sqrt{a^2-b^2}} \tan^{-1} \frac{a \tan \frac{x}{2} + b}{\sqrt{a^2-b^2}} + C, \text{ when } a > b.$$
- $$161. \int \frac{dx}{a+b \sin x} = \frac{1}{\sqrt{b^2-a^2}} \log \frac{a \tan \frac{x}{2} + b - \sqrt{b^2-a^2}}{a \tan \frac{x}{2} + b + \sqrt{b^2-a^2}} + C, \text{ when } a < b.$$

162. $\int \frac{dx}{a^2 \cos^2 x + b^2 \sin^2 x} = \frac{1}{ab} \tan^{-1} \left(\frac{b \tan x}{a} \right) + C.$
163. $\int e^{ax} \sin nx dx = \frac{e^{ax} (a \sin nx - n \cos nx)}{a^2 + n^2} + C.$
164. $\int e^{ax} \cos nx dx = \frac{e^{ax} (n \sin nx + a \cos nx)}{a^2 + n^2} + C.$
165. $\int x e^{ax} dx = \frac{e^{ax}}{a^2} (ax - 1) + C.$
166. $\int x^n e^{ax} dx = \frac{x^n e^{ax}}{a} - \frac{n}{a} \int x^{n-1} e^{ax} dx.$
167. $\int a^{mx} x^n dx = \frac{a^{mx} x^n}{m \log a} - \frac{n}{m \log a} \int a^{mx} x^{n-1} dx.$
168. $\int \frac{a^x dx}{x^m} = -\frac{a^x}{(m-1)x^{m-1}} + \frac{\log a}{m-1} \int \frac{a^x dx}{x^{m-1}}.$
169. $\int e^{ax} \cos^n x dx = \frac{e^{ax} \cos^{n-1} x (a \cos x + n \sin x)}{a^2 + n^2} + \frac{n(n-1)}{a^2 + n^2} \int e^{ax} \cos^{n-2} x dx.$
170. $\int x^m \cos ax dx = \frac{x^{m-1}}{a^2} (ax \sin ax + m \cos ax) - \frac{m(m-1)}{a^2} \int x^{m-2} \cos ax dx;$
 $\int x^m \sin ax = \frac{x^{m-1}}{a^2} (m \sin ax - ax \cos ax) - \frac{m(m-1)}{a^2} \int x^{m-2} \sin ax dx.$

LOGARITHMIC FORMS

171. $\int \log x dx = x \log x - x + C.$
172. $\int \frac{dx}{\log x} = \log (\log x) + \log x + \frac{1}{2^2} \log^2 x + \dots.$
173. $\int \frac{dx}{x \log x} = \log (\log x) + C.$
174. $\int x^n \log x dx = x^{n+1} \left[\frac{\log x}{n+1} - \frac{1}{(n+1)^2} \right] + C.$

$$175. \int e^{ax} \log x dx = \frac{e^{ax} \log x}{a} - \frac{1}{a} \int \frac{e^{ax}}{x} dx.$$

$$176. \int x^m \log^n x dx = \frac{x^{m+1}}{m+1} \log^n x - \frac{n}{m+1} \int x^m \log^{n-1} x dx.$$

$$177. \int \frac{x^m dx}{\log^n x} = -\frac{x^{m+1}}{(n-1) \log^{n-1} x} + \frac{m+1}{n-1} \int \frac{x^m dx}{\log^{n-1} x}.$$

$$\text{APPROXIMATE EVALUATION OF } I = \int_a^b f(x) dx.$$

The interval (ab) is divided into n parts, each equal to Δx .

Let the abscissas of the points of division be $x_0 = a, x_1, x_2, \dots, x_n = b$.

The corresponding values of $f(x)$ are

$$y_0 = f(x_0), y_1 = f(x_1) \dots y_n = f(x_n).$$

Trapezoidal rule (any number of parts)

$$I = \left(\frac{1}{2} y_0 + y_1 + \dots + y_{n-1} + \frac{1}{2} y_n\right) \Delta x.$$

Simpson's (parabolic) rule (even number of parts, n even)

$$I = (y_0 + 4y_1 + 2y_2 + 4y_3 + 2y_4 + \dots + y_n) \frac{\Delta x}{3}.$$

Note. The integrals containing $\sqrt{x^2 + a^2}$ (48–62) can be expressed in terms of the inverse hyperbolic sine by the relation

$$\log \left(\frac{x + \sqrt{x^2 + a^2}}{a} \right) = \sinh^{-1} \frac{x}{a}.$$

The integrals containing $\sqrt{x^2 - a^2}$ (63–77) can be expressed in terms of the inverse hyperbolic cosine by the relation

$$\log \left(\frac{x + \sqrt{x^2 - a^2}}{a} \right) = \cosh^{-1} \frac{x}{a}.$$

DIFFERENTIAL EQUATIONS

1. The differential equation of HARMONIC MOTION.

$$\frac{d^2x}{dt^2} + k^2x = 0.$$

The general solution may be written in the following forms:

$$\begin{aligned} (a) \quad & x = c_1 e^{kt\sqrt{-1}} + c_2 e^{-kt\sqrt{-1}}, \\ (b) \quad & x = A \cos kt + B \sin kt. \end{aligned}$$

We give to (b) another form, thus:

Draw a right triangle with sides A and B . Since A and B are arbitrary constants, this right triangle is arbitrary, and hence also the hypotenuse C and the angle β . Now,

$$A = C \sin \beta, \quad B = C \cos \beta,$$

and substitution in (b) gives

$$x = C (\sin \beta \cos kt + \cos \beta \sin kt), \text{ or}$$

$$(c) \quad x = C \sin(kt + \beta).$$

If in (c) we write for β , $\beta' + \frac{\pi}{2}$, we obtain

$$x = C \sin\left(kt + \beta' + \frac{\pi}{2}\right), \text{ or}$$

$$(d) \quad x = C \cos(kt + \beta').$$

In these formulas c_1 , c_2 , A , B , C , β , β' denote arbitrary constants.

2.

$$\frac{d^2x}{dt^2} - k^2x = 0.$$

The general solution is

$$x = c_1 e^{kt} + c_2 e^{-kt}.$$

3. The differential equation of DAMPED VIBRATION.

$$\frac{d^2x}{dt^2} + 2\mu \frac{dx}{dt} + k^2x = 0, \quad \mu < k.$$

The general solution is

$$\begin{aligned} x &= e^{-\mu t} (A \cos \sqrt{k^2 - \mu^2} t + B \sin \sqrt{k^2 - \mu^2} t), \text{ or} \\ x &= C e^{-\mu t} \cos(\sqrt{k^2 - \mu^2} t + \beta). \end{aligned}$$

4. The differential equation of harmonic motion with a constant disturbing force.

$$\frac{d^2x}{dt^2} + k^2x = c.$$

The general solution is

$$x = A \cos kt + B \sin kt + \frac{c}{k^2}, \text{ or}$$

$$x = C \sin (kt + \beta) + \frac{c}{k^2}.$$

5. The differential equation of FORCED VIBRATION.

$$(a) \quad \frac{d^2x}{dt^2} + k^2x = L \cos nt + M \sin nt, \text{ where } n \neq k.$$

The general solution is

$$x = A \cos kt + B \sin kt + \frac{L}{k^2 - n^2} \cos nt + \frac{M}{k^2 - n^2} \sin nt,$$

where A and B are arbitrary constants.

$$(b) \quad \frac{d^2x}{dt^2} + k^2x = L \cos kt + M \sin kt.$$

The general solution is

$$x = A \cos kt + B \sin kt + \frac{L}{2k} t \sin kt - \frac{M}{2k} t \cos kt.$$

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